THE FUTURE AIR TRANSPORT SYSTEM: LOOKING FOR GENERIC METRICS OF COMPLEXITY FOR TERMINAL AIRSPACE

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ABSTRACT

This paper develops generic metrics for measuring the expected complexity of a given terminal airspace. The metrics include static (infrastructure) and dynamic (traffic on the given infrastructure) complexity, both consisting of the complexity component for arriving and departing traffic. The main objective of the developed metric is as follows: estimation of concepts of airspace organization/design (particular alternative solutions) and related air traffic complexity under given circumstances, which could be used for planning purposes at different time horizons (strategic and tactical).

The main inputs influencing complexity are (both for arriving and departing): traffic demand, runway system capacity, aircraft fleet mix, spatial distribution of traffic, ATC separation rules, number and length of trajectories. Unlike most other approaches, the one presented in this paper excludes the ATC controller from explicit consideration. The outputs consist of complexity values dependant on traffic demand, aircraft fleet mix and the spatial distribution of traffic.

The complexity metric has been illustrated on operations at London Heathrow airport (UK).

KEY WORDS: airport, terminal airspace, complexity metric, modeling

INTRODUCTION

Commercial air transportation demand is forecasted to grow at an average annual rate of 4.5-5% over the forthcoming two decades. Such development is also expected to create additional pressure on the existing and prospective airspace and airport capacity on the one hand, and simultaneously increase air traffic complexity on the other. In order to cope with such development efficiently, effectively, and safely, a new generation of the ATC (Air Traffic Control) ATM (Air Traffic Management) automation, communication, navigation, and surveillance facilities and equipment both airborne and on the ground will need to be developed. During the transition phase from the present to the future system, evaluation of particular alternative solutions will have to be carried out. One component needed for such evaluation will certainly be developing different metrics for estimating concepts of airspace and related air traffic complexity under given circumstances.

In order to fulfill the requirements, these metrics will have to be sufficiently generous in order to be applicable to both existing and future (currently conceptual) solutions. One of these will certainly be the one for measuring airspace and air traffic complexity under conditions of using 3D, 4D and/or the "free- flight" concept *(1)*.

This paper develops such generic metrics for measuring expected complexity of terminal airspace (TMA). Unlike most other approaches, the one presented in this paper excludes the ATC controller (ATCo) from explicit consideration at the initial stage. However, it provides input which could be used for assessment of the ATCo workload under given circumstances, and consequently the ATC unit's capacity *(2)*.

The metrics consist of the indicators of static and dynamic complexity. Being of generic structure, they can handle different shapes of terminal airspace and the arrival and departure trajectories in it, as well as different traffic scenarios along these trajectories and the related airport runway system. Consequently, the output obtained for different terminal airspace layout and traffic scenarios can be used for further estimation of the ATCo workload and consequently the capacity of TMA. Some recent examples of the applicability of the proposed metrics are as follows: changing the runway operating mode at London Heathrow airport (UK) from the present segregated mode (one runway is used exclusively for landings and the other for taking-offs) into the mixed mode (both runways are simultaneously used for both types of operations) *(3)*; and reconfiguring New York low altitude en-route airspace and relocating holding stacks to terminal airspace around the three largest airports *(4)*.

In addition to this introductory section, the paper consists of four other sections. Section 2 describes the characteristics of air traffic in a given terminal airspace and aspects of its complexity requiring the development of generic metrics. Section 3 describes development of complexity metrics for a given terminal airspace. Section 4 presents an application of the proposed metrics of complexity. Finally, the last section summarizes some conclusions and directions for further research.

THE PROBLEM OF AIR TRAFFIC COMPLEXITY IN TERMINAL AIRSPACE

Air Traffic in Terminal Airspace

Terminal airspace represents transitional airspace between airports and ATC sectors. Their size and shape depends on the number of airports contained within it, the airways configurations and the number and length of arrival and departure trajectories. Entry/exit points in/from this airspace are defined by radio-navigation aids. In some cases these points serve as an entry point for holding stacks. It contains arrival trajectories converging on the airport and departure trajectories diverging from it. Aircraft traverse the TMA at a broad range of speeds: after take-off they accelerate, increasing their altitude, and during approach decelerate decreasing altitude. Flight through the TMA is possible in one of the following ways *(5, 6)*:

- On trajectories defined by courses, distances and altitudes between radio-navigation aids. Trajectories defined in such a way are called STARs (Standard Terminal Arrival) and SIDs (Standard Instrument Departure).
- Defining 2D, 3D and 4D trajectories by Area Navigation (RNAV) or Required Navigation Performance (RNP) methods.
- Executing instructions given by ATCo's (radar vectoring) consisting of a series of vectors containing courses, points and altitudes over specific points.

TMA is a highly complicated system that is very sensitive to changes (traffic, meteorological, technical, procedural, administrative, etc.). All changes, regardless of their nature, influence the state of the system which is (by nature) dynamic. Measuring the system state is becoming an important problem that mostly depends on system complexity.

Complexity of Air Traffic

ATM is a hierarchical process where activity on each level is meant to produce a less complex output situation, from its input situation (origin-destination matrix), and which is to be handled by the next level. Air Traffic Complexity may be reduced at the strategic, (pre) tactical and operational level. At each of these levels it can have a spatial based nature such as air space and airfield system design and/or assignment (air routes, sectors, terminals, runway systems, etc.), but also time based solutions (schedules, slot allocations, flow management, etc). It is understood as a demand characteristic of traffic that is to be served by an appropriate supply system, or modified in such a way as to make it possible to serve it using an existing supply system *(7)*.

According to *(8)* no single universal complexity measure exist other than a set of complexity metrics, useful and relevant in a particular context and for a particular purpose. In this paper, the complexity of air traffic is defined as a measure of quantity as well as of quality (characteristics) of interactions between flights *(7, 6)*.

Some Previous Research

Over the last decade, concern about the problems of measuring the difficulty of a traffic situation, i.e. it's complexity, has risen. Authors in *(9)* were some of the first to deal with complexity and its influence on ATCo workload. They identified two basic elements of ATC complexity: sector complexity and traffic complexity. Dealing with "free flight" influence on physical and mental workload of ATCo, Pawlak et al. *(10)* developed a model of air traffic complexity with the hypothesis that complexity causes complete change in ATCo cognitive workload.

In order to measure ATCo workload *(11)* introduced a concept called "dynamic density" (DD) which induces traffic density (number of aircraft in airspace) and traffic complexity (measure of traffic complexity in airspace). DD was applied in *(12)* with aim of determining whether a given measure could be predicted in the future. The DD concept was further elaborated and its applicability further broadened in papers *(13, 14, 15, 16)*. Schaefer (*17)* defines complexity as a measure of the difficulty that a particular traffic situation will present to an ATCo. This measure is limited to the characteristics of the traffic situation itself, and may thus be considered a factor which causes workload. In this paper, complexity was used as a key concept for solving the problem of sector capacity and controller workload. Authors in *(18, 19, 20)* dealt with the problem of measuring complexity of air traffic. They assumed that airspace complexity is related to the traffic structure and airspace geometry. According to this assumption they concluded that a measure of complexity would find wide application in: balancing with sector load, distribution of traffic in the sense of congestion, new airways network design, dynamic sectorization, slot allocation, traffic flow management, comparison of different airspace structure effectiveness etc. Following previous work *(2, 8)* applied complexity metrics for airspace configuration.

Other approaches to define, measure or even reduce air traffic or airspace complexity have recently appeared, opening a new field for the further application of the complexity metric.

It can be concluded that great attention was given to modeling and measuring complexity in the en-route environment related to ATCo workload. TMA traffic complexity has received a moderate degree of attention. Measures of traffic complexity in TMA are presented in the work of *(7, 6)*. They develop simulation models for instantaneous measurement of traffic complexity. This paper presents a further elaboration of TMA traffic complexity and development of analytical models for complexity assessment.

MODELING COMPLEXITY METRICS FOR TERMINAL AIRSPACE

The Main Objectives and Assumptions

The objective of modeling the expected complexity of a given TMA is to develop a metric for estimating concepts of airspace organization/design (particular alternative solutions) and related air traffic complexity under given circumstances, which could be used for planning purposes at different time horizons, mainly strategic (e.g. introduction of a new runway at an existing airport or a new airport in the TMA) and tactical (in sense of using different tactics, i.e. runway operating modes, TMA entry/exit points, etc.).

Similarly with the past research, the main assumption in this paper is that complexity depends on airspace geometry (static element) and air the traffic using it (dynamic element). Therefore, complexity metrics consist of the indicators of static and dynamic complexity. Being of generic structure, they can handle different TMA shapes and the arrival and departure trajectories in it, as well as different traffic scenarios along these trajectories and the related airport runway system. The indicator of static complexity implies the number and the 3D shape of the STARs and SIDs in a given TMA. The indicator of the dynamic complexity for a given TMA implies the number and character of the prospective interactions (potential conflicts during a given period of time) between the arriving and/or departing aircraft while being on the particular STARs and SIDs, respectively. The proposed metrics are expressed in the relative terms, which enable comparison of complexity of different TMAs. Unlike past research approaches, the approach in this paper excludes the ATC workload from an explicit consideration, but provides an input for its estimation depending on the TMA design and the related traffic scenarios.

Because of their inherently generic structure, complexity metrics could be used for:

- Planning purposes at strategic and tactical level, i.e. initial assessment of the complexity of the current, transitional, and future TMA after slight modifications (in the process of re-planning and re-design of a given airspace or re-activating currently military airspace for commercial flights); and
- Evaluation of the technical/technological feasibility of alternative airspace design, supported by particular technologies.

In developing indicators of static and dynamic complexity, the following assumptions are introduced:

- The complexity values are actually expected one;
- The particular parameters influencing the particular factors of the static and dynamic complexity of a given TMA are constant in the specified period of time (half an hour, one hour);
- The arriving and departing aircraft continuously and simultaneously change altitudes while flying through a given TMA, which requires application of the radar distance-based separation rules;
- The runway system capacity is always allocated in proportion to the intensity of traffic on particular arrival and departure trajectories;
- The runway system is operated by always assigning ultimately higher priority to landings, rather than taking-offs, if they both simultaneously request a service.

The Models of Complexity Metrics

Static complexity metrics

The indicator of the static complexity of a given TMA can be considered as consisting of two components; the component C_{sa} for the arriving traffic, and the component C_{sd} for the departing traffic. Each component is expressed by the product of two factors: i) the factor of external complexity; and ii) the factor of internal complexity.

The former factor aims at expressing the 3D spatial size of the TMA used for setting up the arrival and/or the departure trajectories. The latter factor expresses the internal characteristics of the geometry of each type of these trajectories and their mutual spatial interactions.

Consequently, the indicator of the static complexity of a given TMA can be expressed as:

$$CS = C_{sa} + C_{sd} = \alpha_{sa}\beta_{sa} + \alpha_{sd}\beta_{sd} \quad (1)$$

where

α_{sa}, $\alpha_{sd.}$ is the factor of the external complexity for the arriving and departing traffic, respectively;

β_{sa}, β_{sd} is the factor of the internal complexity for the arriving and departing traffic, respectively;

The indicator *CS* is not dimensional like the factors α and β.

Under the assumption that each arrival and departure trajectory in a given TMA can be approximated by an extended pyramid with a rectangular basis at the TMA's entry/exit points and a cone at the FAG (Final Approach Gate) for the arrival trajectories and the runway departure threshold for the departure trajectories, the factors α_{sa} and α_{sd} can be estimated as follows:

$$\alpha_{sa} = \frac{1/3\sum_{k=1}^{M} 6\sigma_{xy/k}\, 6\sigma_{z/k}\, d_k}{\phi_a PH} \quad (2a)$$

and

$$\alpha_{sd} = \frac{1/3\sum_{m=1}^{N} 6\sigma_{xy/m}\, 6\sigma_{z/m}\, d_m}{\phi_d PH} \quad (2b)$$

where

$\sigma_{xy/k}$, $\sigma_{z/k}$ is the standard deviation of the actual from the prescribed arrival trajectory *(k)* in the horizontal and the vertical plane, respectively (nm);

$\sigma_{xy/m}$, $\sigma_{z/m}$ is the standard deviation of the actual from the prescribed departure trajectory *(m)* in the horizontal and the vertical plane, respectively (nm);

d_k, d_m is the length of the arrival trajectory *(k)* and the departure trajectory *(m)*, respectively, measured in the horizontal plane (nm);

P is the area of a given TMA in the horizontal plane (nm^2);

H is the vertical difference between the maximum and minimum altitude embraced by a given TMA (feet);

ϕ_a, ϕ_d is the portion of the total volume of a given TMA in which the arrival and the departure trajectories spread, respectively.

M, N is the number of the arrival and the departure trajectories, respectively.

In the expression (2), the sum of the proportions $\phi_a + \phi_d$ does not need to be equal to one if the particular arrival and departure trajectories share the same portion of the TMA. In addition, it implies that, if 99% of the actual arrival and/or departure trajectory lie in a given corresponding pyramid, its basis as a rectangle will have a horizontal side of length of 2x3$\sigma_{xy/.}$ and a vertical side of length of 2x3$\sigma_{z/.}$.

Under the assumption that the smallest complexity will be achieved in a TMA with straight approach/departure as well as continuous descent approach, the factors β_{sa} and β_{sd} in the expression (1) can be determined as follows:

$$\beta_{sa} = \sum_{k=1}^{M}\left[\sum_{s=1}^{w_k}\varphi_{k/s} + \sum_{r=1}^{h_k}\theta_{k/r} + \sum_{\substack{l=1 \\ l\neq k}}^{M} n_{kl} + \sum_{m=1}^{N} n_{km}\right] \tag{3a}$$

and

$$\beta_{sd} = \sum_{m=1}^{N}\left[\sum_{s=1}^{w_m}\varphi_{m/s} + \sum_{r=1}^{h_m}\theta_{m/r} + \sum_{k=1}^{M} n_{mk}\right] \tag{3b}$$

where

w_k, h_k is the number of changes of heading and altitude, respectively of the arrival trajectory *(k)*;

n_{kl} is the number of times the arrival trajectory *(k)* crosses and/or merges with the arrival trajectory *(l)*;

$\varphi_{k/s}$ is an integer for *(s)*-st change of heading along the arrival trajectory *(k)*; it takes the value increased by "1" for each change of heading for additional 90^0;

$\theta_{k/r}$ is an integer for *(r)*-th change of altitude along the arrival trajectory *(k)*; it takes the value increased by "1" for each change of altitude for an additional 2000 ft;

n_{km} is the number of times the arrival trajectory *(k)* crosses the departure trajectory *(m)*;

w_m, h_m is the number of changes of heading and altitude, respectively, of the departure trajectory *(m)*;

$\varphi_{m/s}$ is an integer for *(s)*-st change of heading along the departure trajectory *(m)*; it takes the value increased by "1" for each change of heading for an additional 90^0;

$\theta_{m/r}$ is an integer for *(r)*-th change of altitude along the departure trajectory *(k)*; it takes the value increased by "1" for each change of altitude for an additional 2000 ft;

n_{mk} is the number of times the departure trajectory *(m)* crosses the arrival trajectory *(k)*.

Dynamic complexity metrics

Background In general, the indicator of the dynamic complexity in a given TMA consists of two sub-indicators: for the arriving and for the departing traffic. Each of these can be expressed by the factor reflecting the general traffic load along particular arrival and/or departure trajectories and the interactions between aircraft on these trajectories. In the latter case the aircraft on the same arrival and/or the same departure trajectory may have some interactions as well as the aircraft on an arrival and other departure trajectory, and vice versa.

Arriving traffic The sub-indicator of the dynamic complexity for the arriving traffic can be determined as the sum of the factors reflecting the general arriving traffic complexity $C_{a/g}$, the potential overtaking conflicts along the same arrival trajectories $C_{a/o}$, the potential crossing conflicts on the different crossing and/or merging points of particular arrival trajectories $C_{a/c}$, and the potential crossing conflicts between the traffic on particular arriving and the traffic on particular (crossing) departing trajectories $C_{a/d}$ as follows:

$$CD_a = C_{a/g} + C_{a/o} + C_{a/c} + C_{a/d} \tag{4}$$

The factor of general complexity $C_{a/g}$ This factor reflects the arriving traffic load in a given TMA, during a given period of time. Under the assumption that the arrival capacity of the runway system C_a is allocated proportionally to the intensity of traffic on each arrival trajectory, this indicator can be estimated as:

$$C_{a/g} = \lambda_a / C_a \tag{5}$$

where

λ_a is the intensity of arriving traffic in a given TMA (ac/unit of time);

The factor of potential overtaking conflicts $C_{a/o}$ This factor implies estimation of the number of potential overtaking conflicts, the maximum number of potential interactions between aircraft on the given arrival trajectories, and the ratio between the previous two.

a) The number of potential overtaking conflicts along the arrival trajectory *(k)* can be estimated as follows *(21)*:

$$C_{ok} = (\lambda_a q_{ak})^2 d_k \sum_{ij,i<j}^{I_k} \left[1/v_{aki} - 1/v_{akj}\right] p_{aki} p_{akj} \quad (\text{ac}^2/\text{unit of time}) \tag{6a}$$

where

q_{ak} is the proportion of the arriving traffic using the arrival trajectory *(k)*;

d_k is the length of the arrival trajectory *(k)* (nm);

I_k is the number of different aircraft categories using the arrival trajectory *(k)*;

v_{aki}, v_{akj} is the average arrival speed of the aircraft category *(i)* and *(j)*, respectively, along the trajectory *(k)* (nm/unit of time);

p_{aki}, p_{akj} is the proportion of the aircraft category *(i)* and *(j)*, respectively, on the arrival trajectory *(k)*.

In addition:

$$\sum_{k=1}^{M} q_{ak} = 1, \quad \sum_{i=1}^{I_k} p_{aki} = 1 \quad \text{for } \forall k \in M \quad \text{and} \quad \lambda_a \le C_a \tag{6b}$$

The factor C_{ok} is expressed in (ac^2/unit of time), which is actually the intensity of the number of potential conflicts per unit of time. Furthermore, the traffic intensity λ_a and the probabilities q and p can both change over time. Last but not least, if $\lambda_a = C_a$, the number of potential overtaking conflicts will be maximal, all other parameters fixed.

b) The maximum number of possible interactions along the arrival trajectory *(k)* can be approximated as:

$$C_{ak/\max} = (C_a q_{ak})^2 \left[d_k / \sum_{i=1}^{I_k} p_{aki} v_{aki} \right] \quad (\text{ac}^2/\text{unit of time}) \tag{6c}$$

where all symbols are as in the previous expressions.

c) Using the expression (6a-c), the factor of complexity due to potential overtaking conflicts Ca/o can be estimated as:

$$C_{a/o} = \sum_{k=1}^{M} C_{ok} / C_{ak/\max} \tag{6d}$$

The factor of potential crossing conflicts $C_{a/c}$ This factor implies estimation of the number of prospective crossing conflicts, the maximum number of aircraft interactions, and the ratio between the previous two.

a) The number of prospective crossing conflicts between the arrival traffic along the trajectory *(k)* and the arrival traffic along the trajectory *(l)* can be determined as:

$$C_{ac/kl/p} = (\lambda_a^2 q_{ak/p} q_{al/p}) t^*_{akl/p} P(T_{akl/p} \le t^*_{akl/p}) \quad (\text{ac}^2/\text{unit of time}) \tag{7a}$$

where

λ_a is the intensity of the arriving traffic in a given TMA (ac/unit of time);

$q_{ak/p}$, $q_{al/p}$ is the proportion of the arriving traffic using the arrival trajectory *(k)* and the arrival trajectory *(l)*, respectively, at the crossing and/or merging point *(p)* of these trajectories;

$t_{akl/p}^*$ is the minimum prescribed average time interval between passing an arriving aircraft on the trajectory *(k)* and an arriving aircraft on the trajectory *(l)* through the crossing and/or merging point *(p)* of these trajectories (time units);

$P(T_{akl/p} \leq t^*_{akl/p})$ is the probability that the time separation between passing of the arriving aircraft on the trajectory *(k)* and the arriving aircraft on the trajectory *(l)* through the crossing and/or merging point *(p)* of these trajectories, as the stochastic variable $T_{akl/p}$ takes the value(s) less than the minimum prescribed time separation $t_{akl/p}^*$;

In the expression (6a), it is most reasonable to assume that the stochastic variable $T_{akl/p}$, generally influenced by many factors, has the Gaussian probability function with the known mean and standard deviation, and that it is not dependant on demand/capacity ratio;

The minimum time $t_{akl/p}^*$ can be determined as follows *(21)*:

$$t^*_{akl/p} = \sum_{i=1}^{I_k}\sum_{j=1}^{I_l} p_{aki/p} t^*_{akilj/p} p_{alj/p} = \sum_{i=`1}^{I_k}\sum_{j=1}^{I_l} p_{aki/p} \frac{\delta_{akilj/p}[v^2_{aki/p} - 2v_{aki/p}v_{alj/p}\cos\alpha_{kl/p} + v^2_{alj/p}]^{1/2}}{v_{aki/p}v_{alj/p}\sin\alpha_{kl/p}} p_{alj/p} \tag{7b}$$

where

$\delta_{akilj/p}$ is the minimum distance separation between the leading aircraft *(i)* on the arrival trajectory *(k)* and the trailing aircraft *(j)* on the arrival trajectory *(l)* during their passing through the trajectories' crossing or merging point *(p)* at the moment when they are the closest to each other (nm);

$v_{aki/p}$, $v_{alj/p}$ is the average arrival speed of the aircraft category *(i)* and *(j)*, respectively, while passing through the crossing or merging point *(p)* of the arrival trajectories *(k)* and *(l)* (nm/unit of time);

$p_{aki/p}$, $p_{alj/p}$ is the proportion of the aircraft category *(i)* and *(j)* on the crossing (or merging) point *(p)* of the arrival trajectories *(k)* and *(l)*, respectively;

I_k, I_l is the number of different aircraft categories using the arrival trajectory *(k)* and *(l)*, respectively.

The total number of potential crossing conflicts between the arriving flows on the trajectories *(k)* and *(l)* can be determined based on (7a-b) as follows:

$$C_{ac/kl} = \sum_{p=1}^{n_{kl}} C_{ac/kl/p} \tag{7c}$$

where

n_{kl} is the number of merging (crossing) points between the arrival trajectory *(k)* and the arrival trajectory *(l)* (it is specified as n_{ij} in the expression (3a).

The total number of potential crossing conflicts between all combinations of the arrival trajectories in a given TMA can be estimated as:

$$C_{ac} = \sum_{k=1}^{M}\sum_{l=1}^{M}\sum_{p=1}^{n_{kl}} C_{ac/kl/p} \text{ for } k \neq l \tag{7d}$$

b) The maximum possible number of crossing interactions can be determined as:

$$C_{ac/\max} = C_a^2 \sum_{k=1}^{M}\sum_{l=1}^{M}\sum_{p=1}^{n_{kl}} (q_{ak/p} T_{akl/p} q_{al/p}) \text{ for } k \neq l \tag{7e}$$

The value $T_{akl/p}$ in the expression (7e) is usually selected as the difference between the maximum and the minimum value of the corresponding stochastic variable, thus implying the probability of its realization to be equal to one. Also in this case, it is reasonable to assume that the stochastic variable $T_{akl/p}$ has Gaussian density function with given mean and standard deviation, and that it is not dependant on demand/capacity ratio.

c) Using the expressions (7d-e), the factor of complexity due to potential crossing conflicts in a given TMA can be estimated as:

$$C_{a/c} = C_{ac} / C_{ac/\max} \tag{7f}$$

The factor of potential arrival/departure conflicts $C_{a/d}$ With appropriate modifications of particular subscripts to indicate the arrival and departure trajectory and related traffic, the analogous expressions to (7a-f) can be used to estimate the factor of complexity $C_{a/d}$. Consequently, the number of potential crossing conflicts between all combinations of the arriving and the departing trajectories between which the crossings exist can be determined as follows:

$$C_{ad/c} = \sum_{k=1}^{M}\sum_{l=1}^{M}\sum_{p=1}^{n_{kml}} C_{ad/c/kl/q} \tag{8a}$$

The maximum number of possible interactions between the arriving and departing traffic at the crossing points of the related trajectories can be estimated similarly as:

$$C_{ad/c/\max} = C_a C_d \sum_{k=1}^{M}\sum_{m=1}^{N}\sum_{q=1}^{n_{km}} (q_{ak/q} T_{ad/kl/q} q_{dk/q}) \tag{8b}$$

where

C_d is the departure capacity of the runway system (ops/unit of time)

Similarly as in the expression (7e), the value $T_{ad/kl/q}$ in the expression (8b) is usually taken as the difference between the maximum and the minimum value of the corresponding stochastic variable with the assumed Gaussian probability distribution, thus implying its probability of realization to be equal to one. Consequently, the factor of complexity due to potential conflicts between the arriving and departing traffic can be estimated from the expressions (8a-b) as follows:

$$C_{a/d} = C_{ad/c} / C_{ad/c/\max} \tag{8c}$$

Departing traffic The sub-indicator of the dynamic complexity for the departing traffic can be determined as the sum of the factors reflecting the general departing traffic complexity $C_{d/g}$, the potential overtaking conflicts along the same departure trajectories $C_{d/o}$, and the potential crossing conflicts between the traffic on particular departing and the traffic on particular (crossing) arriving trajectories $C_{d/a}$, as follows:

$$CD_d = C_{d/g} + C_{d/o} + C_{d/a} \tag{9}$$

The factor of general complexity $C_{d/g}$ Analogously as in expression (5), the factor $C_{d/g}$ can be estimated, under the assumption that the runway system departure capacity is allocated proportionally to the intensity of the traffic on each departure trajectory, as follows:

$$C_{d/g} = \lambda_d / C_d \tag{10}$$

where

λ_d is the intensity of departing traffic in a given TMA (ac/unit of time);

The factor of potential overtaking conflicts $C_{d/o}$ This factor can be estimated after adjusting particular symbols in the expression (6a-d) and by using similar reasoning. In this case, the length of the departure trajectories should be modified bearing in mind that after some time the vertical separation between departing aircraft can be established and the potential for overtaking conflicts thus may disappear.

The factor of potential departure/arrival conflicts $C_{d/a}$ This factor can be estimated, after appropriate adjustments of symbols, as in the expressions (8a-c).

The use of the same runway system

The specific sub-indicator of dynamic complexity relates to the potential interactions between the landing and taking-off aircraft during use of the same system of dependent runways (single, close parallel, and/or intersecting runways). Under the assumption that landings always have the ultimate priority over taking-offs, when both occur simultaneously, their prospective interactions can be expressed as follows:

$$C_{ad/r} = \varepsilon\left[1 - p_d(\lambda_a)\right] \text{ for } C_a^* \le \lambda_a \le C_a \tag{11a}$$

where

ε is a binary variable, which takes the value "1" if the landings and take-offs are carried out on the dependent runways, and the value "0", otherwise;

$p(\lambda_a)$ is the probability of time-space gaps occurring between the successive landings of intensity λ_a, which allow a safe realization of at least one take-off between them;

C_a^* is the landing capacity of the runway system, which enables, realization of at least one take-off between each pair of the successive landings, i.e. $p(C_a^*) = 1$

The capacities C_a^* and C_a can be determined as follows *(22)*:

$$\lambda_a^* = 1/\bar{t}_{a/d} \tag{11b}$$

and

$$C_a = 1/\bar{t}_a \tag{11c}$$

where

$\bar{t}_{a/d}$ is the average time, which enables safe realization of at least one take-off between each pair of successive landings;

$\bar{t}_a$ is the minimum average inter-arrival time between successive landings, which enables realization of take-off only between pairs of successive landings where the separation safety conditions are fulfilled.

The condition, which allows safe take-off(s) between a pair of successive landings *(ij)*, can be estimated as follows *(22)*:

$$t_{ij/d} = \overline{R_i} + \overline{\delta_j} / \overline{v_j} + (n-1)t_{dk} \tag{11d}$$

were:

$\overline{R_i}$ is the average runway landing occupancy time by the leading aircraft *(i)* in the landing sequence *(ij)*;

$\overline{\delta_j}$ is the minimum average separation distance between the landing aircraft *(j)* in the landing sequence *(ij)* and an aircraft taking-off measured along the path of the former one;

$\overline{v_j}$ is the average approach speed of the landing aircraft *(j)* along the distance $\overline{\delta_j}$;

n is the number of successive taking-offs between the landings *(i)* and *(j)*;

t_{dk} is the runway occupancy time by the aircraft taking-off *(k)*.

The overall dynamic complexity

Based on the above mentioned reasoning, the indicator of the dynamic complexity for a given TMA, traffic scenario, and period of time, can be estimated from the expressions (4), (9) and (11), as follows:

$$CD = CD_a + CD_d + C_{ad/r} \tag{12}$$

The total complexity of a given TMA

Using the expressions (1) and (12), the total complexity of a given TMA can be estimated as the sum of static and the dynamic complexity as follows:

$$CT = CS + CD \tag{13}$$

AN APPLICATION OF THE MODELS

Description of Input

The model is applied to London Heathrow airport (UK). This appears to be a convenient case since the airport operates close to saturation under rather strict constraints in terms of noise and in the vicinity of the four other London airports *(3)*. The airport's two - parallel runway system currently operates in the segregated mode implying that one runway is always dedicated to arrivals and another to departures for both Westerly (WOPS) and Easterly (EOPS) operations. On the former, used approximately 70% of the year, landings are carried out on the runway 27R and take-offs on the runway 27L. On the latter, landings are carried out on the runway 9L and take-offs on the runway 9R (Figure 1) *(23)*.

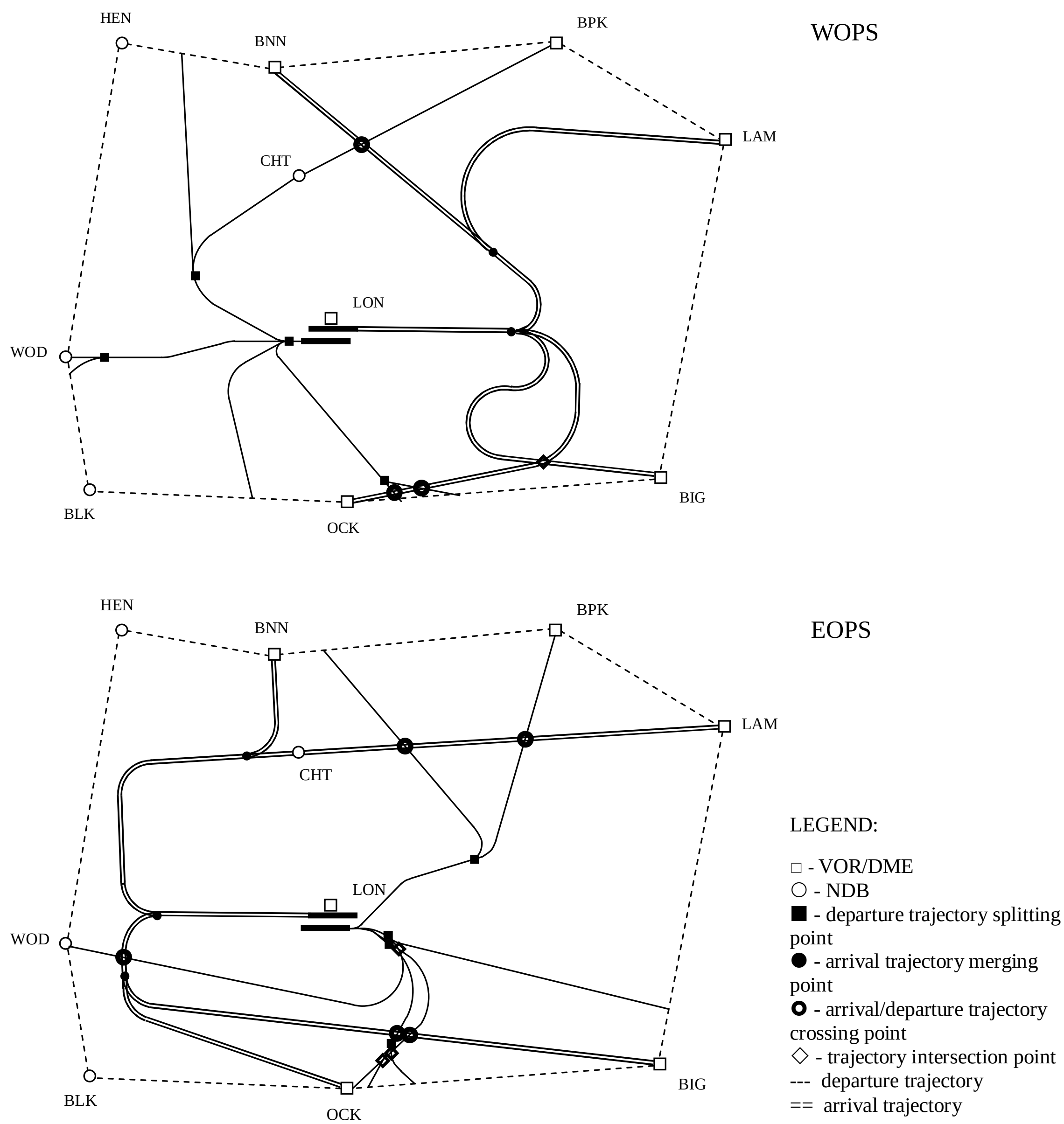

FIGURE 1 London Heathrow TMA, arrival and departure trajectories (Compiled from: *24*)).

The airport runway system capacity for arrivals and departures is 44 ops/h each while operating in the segregating mode *(25)*. The structure of the aircraft fleet mix is: p_i = 0.65 (Medium jets) and p_j = 0.35 (Heavy jets). Their average arrival speeds through TMA is v_i = 220kt and v_j = 240kt, respectively. The ATC average separation rule is: δ = 3.5nm. The arriving/departing traffic is assumed to be equally distributed among the northern and the southern entry and exit points. For illustration purposes only arrivals were chosen.

Analysis of the Results

In order to investigate the dependence of a particular complexity metric of Dynamic complexity for arrivals CD_a on the particular influencing parameters, sensitivity analysis was carried out with respect to: a) the demand/capacity ratio (λ_a/C_a); b) the aircraft fleet mix; and c) distribution of traffic among the TMA entry points. The results are shown in Figures 2, 3, and 4.

Figure 2 represents dependence of Dynamic complexity CD_a on the demand/capacity ratio (λ_a/C_a). Changes of particular complexity components are also shown.

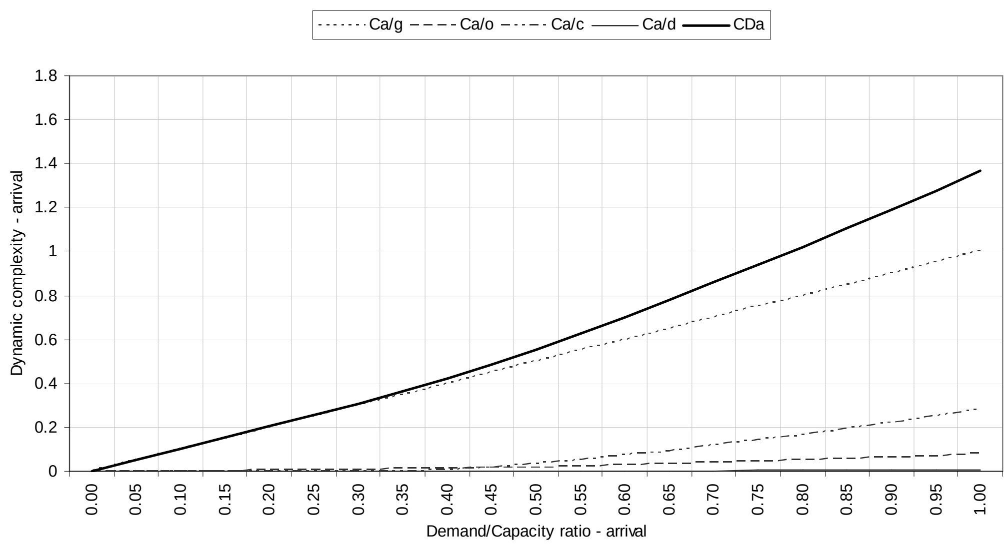

FIGURE 2 Dependence of Dynamic complexity CD_a on the demand/capacity ratio (λ_a/C_a).

As can be seen, the values of CD_a increase more than proportionally with the increase of the ratio λ_a/C_a. The maximum value is CD_a = 1.37 for λ_a/C_a = 1. The main contributing factor is the complexity component $C_{a/g}$ with 73%, i.e. 1 out of 1.37. $C_{a/g}$ is changing in proportion and mostly influencing CD_a. Other influencing components are $C_{a/c}$ with a share of 20.5%, $C_{a/o}$ with a share of 6% and $C_{a/d}$ with the share of only 0.5%.

Figure 3 shows the dependence of Dynamic complexity CD_a on the aircraft fleet mix and the different values of the demand/capacity ratio (λ_a/C_a): 1, 0.75, 0.5, and 0.25. As can be seen, Dynamic complexity CD_a increases with increases in the demand/capacity ratio, while reaching maximum values for the maximum heterogeneity of the fleet mix (50:50 Heavy vs. Medium jets). The incremental increase in CD_a is higher when the demand/capacity ratio approaches the value of 1. The maximum CD_a for analyzed cases are: 1.37, 0.94, 0.55, and 0.26. The minimum values occur when the homogeneity of the aircraft fleet (with only Heavy jets) is maximal.

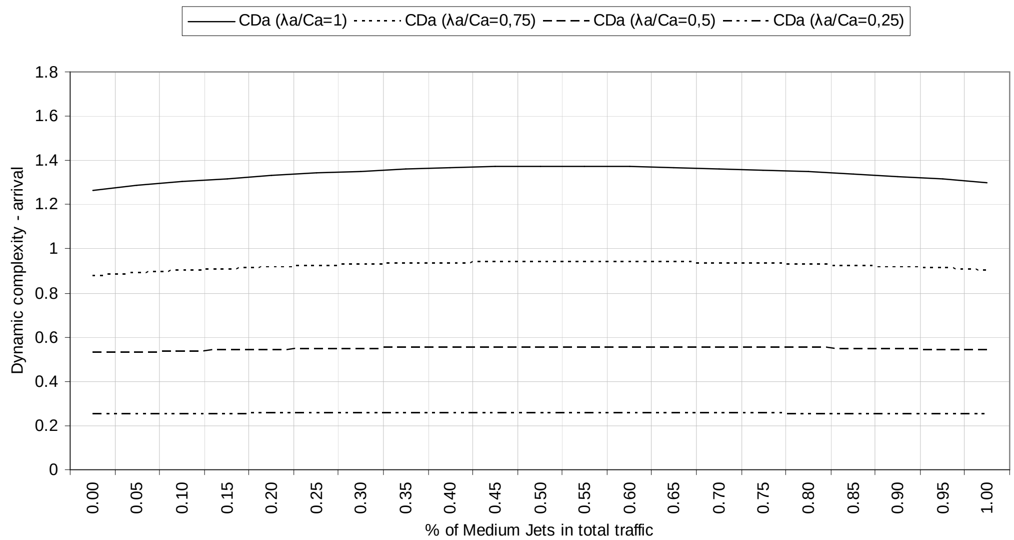

FIGURE 3 Dependence of Dynamic complexity CD_a on the aircraft fleet mix and the demand/capacity ratio (λ_a/C_a).

Figure 4 shows the dependence of Dynamic complexity CD_a on the distribution of traffic among particular TMA entry points and the demand/capacity ratio (λ_a/C_a). As in the previous examples, four different values for the demand/capacity ratio are used: 1, 0.75, 0.5, and 0.25. As can be seen, Dynamic complexity CD_a increases with increases in the demand/capacity ratio while being strongly dependent on the traffic distribution between the TMA entry points. For example, it is lower when the northern traffic becomes dominant. Consequently, the minimum values for CD_a are: 1.14 for 80%, 0.82 for 75%, 0.52 for 70%, and 0.26 for 50% of the arriving traffic from the north, respectively. The maximum values for CD_a occur when the traffic arrives only from the south. The main reason for such an outcome lies in the shape and the mutual relationships between the arrival trajectories from the south (from the entry points OCK and BIG (see Figure 1)) as well as the arrival and departure trajectories.

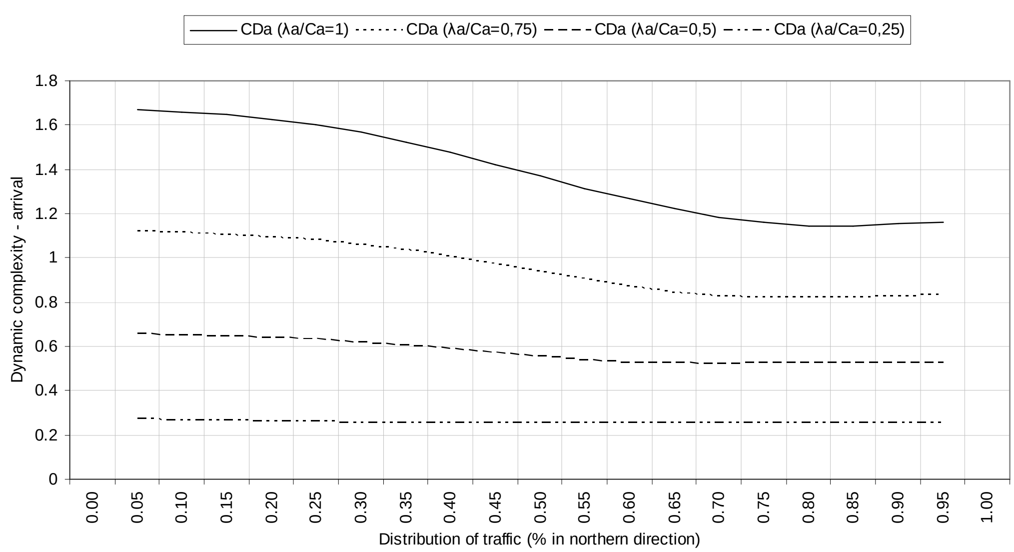

FIGURE 4 Dependence of Dynamic complexity CD_a on the distribution of traffic between TMA entry points and the demand/capacity ratio (λ_a/C_a).

In order to show how the developed complexity metric could possible be applied, a comparison of Dynamic complexity in case of arrivals CD_a for WOPS and EOPS was performed. Comparison can generally indicate which operations are more complex. It is made relative to changes of traffic distribution (Figure 5).

Figure 5 represents the dependence of Dynamic complexity for the arriving traffic CD_a on distribution of traffic – WOPS vs. EOPS. Complexity values in case of WOPS are higher than those for EOPS up to 80% of traffic from the north. After that, the complexity value for EOPS becomes slightly higher. Comparing the two curves it is evident that the deviation of the complexity values in the case of EOPS is much lower than in that of WOPS. Furthermore, it is not evident which traffic flow is dominant in the case of EOPS. The minimum complexity value in the case of EOPS is obtained for equally distributed traffic between north and south. The possible reason for such a result may lie in the shape and length of arrival trajectories as well as intersection between them and between arrival/departure trajectories (Figure 1).

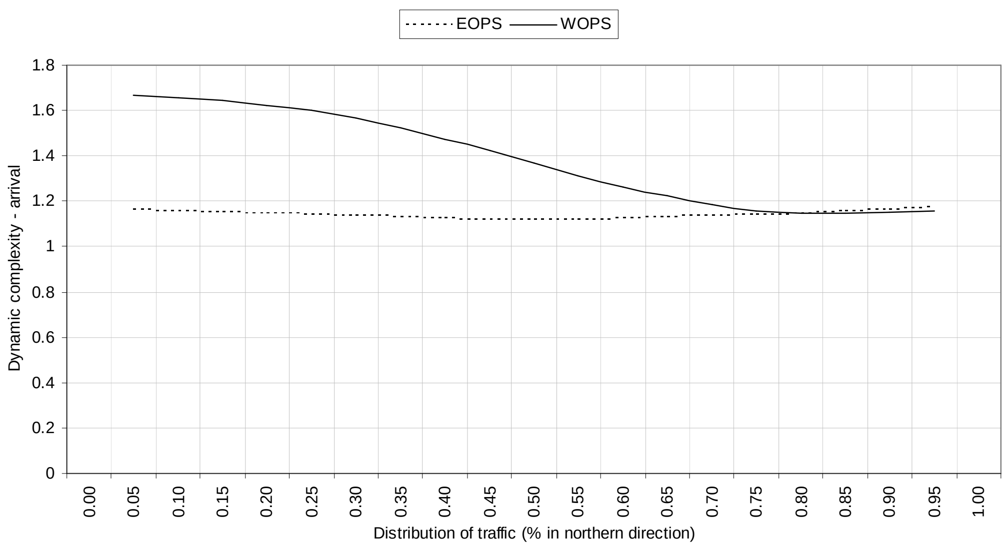

FIGURE 5 Dependence of Dynamic complexity for the arriving traffic CD_a on distribution of traffic – WOPS vs. EOPS.

CONCLUSIONS

The paper has presented generic metrics for measuring the expected complexity of a given terminal airspace. The metrics include static and dynamic complexity, both consisting of a complexity component for arriving and departing traffic.

The complexity metric was applied to the arrivals at London Heathrow airport (UK). The results of a Westerly Operations scenario have shown that the dynamic complexity increases more than linearly with increasing traffic intensity. In addition, the maximum complexity occurs with maximum heterogeneity of the aircraft fleet thus implying that a more homogeneous aircraft fleet contributes to reducing complexity. Comparisons between Westerly and Easterly operations have shown that Easterly are not sensitive, like Westerly, to traffic distribution changes. It could be expected that overall complexity of Heathrow TMA would significantly increase adding the complexity for departures, although it is not estimated in this paper.

Due to its generosity, the developed metric could also be applied for the existing, modified, and new TMA layouts around a single airport or a close group of airports for different planning levels (strategic and tactical) and time horizons (short- and long-term). This also includes the introduction of a new runway, with corresponding departing and arriving trajectories and modification of the existing trajectories due to other constraints.

The metric could also be of assistance in assessing the complexity of new traffic scenarios for London Heathrow airport if it operated in the mixed mode and if a new third parallel runway is constructed. Other challenging cases are airports with intersecting or converging runways (e.g. Zurich airport) or multi airport TMA systems (e.g. New York TRACON).

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