



## MODELIRANJE IZBORA SAOBRAĆAJNOG TOKA ZA POTREBE PLANIRANJA U KONTROLI LETENJA

### MODELLING OF FLOW CHOICE FOR AIR TRAFFIC CONTROL PLANNING PURPOSES

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**Rezime:** U radu je predstavljen model izbora saobraćajnog toka, razvijen za potrebe planiranja u Kontrolu letenja. Zasnovan je na Agregiranom Multinomijalnom LOGIT modelu i sadrži dva "otpora" koji utiču na izvršenje letova, tj. izbor toka. Otpori su predstavljeni kao kombinacija dužine toka, jedinične cene preleta i broja vazdušnih puteva u toku, posmatranog vazdušnog prostora. Model je razvijen za potrebe predviđanja obima saobraćaja po tokovima koji zavisi od promene jediničnih cena preleta i kvaliteta usluge u posmatranom i okolnim vazdušnim prostorima. U radu je, na primeru Agencije za Kontrolu letenja Srbije i Crne Gore, prikazan proces modeliranja: izbor podataka, izbor varijabli, estimacija parametara i validacija modela.

KLJUČNE REČI: IZBOR TOKA, LOGIT MODEL, KONTROLA LETENJA

**Abstract:** A Traffic Flow Choice model, developed for Air Traffic Control planning purposes, is presented in this paper. It is based on Aggregate Multinomial LOGIT model and contains two "resistances" which are influencing flight behaviour, i.e. choice of flow. Resistances in considered airspace are presented as combinations of flow length, unit over flight charge (unit rate) and number of airways in flow. Model is developed for prediction of traffic throughput per flow depending on changes of unit rate and quality of service in given and surrounding airspaces. Paper presents description of modelling process in case of Serbia and Montenegro Air Traffic Services Agency: data choice, variable choice, parameters estimation as well as model validation.

KEY WORD: FLOW CHOICE, LOGIT MODEL, AIR TRAFFIC CONTROL

## 1. INTRODUCTION

During the last decade, traffic over Serbian and Montenegrin (SM) airspace was reduced and rerouted on airspaces of surrounding countries (Romania, Croatia, and Italy). In last few years, traffic volume was recovering due to opening of SM airspace and introduction of new airways. But, open issue remain, what more could be done in future to secure steady traffic development in SM airspace? This paper present the model designed for estimation of possibilities for traffic attraction in situation of multiple competitions, when traffic could be attracted by better quality of service and lesser unit over flight charge (unit rate) offered to the users. Process of flow choice modelling is presented, consisting of several steps presented in following sections: choice of data which will be used for modelling; choice of explanatory variables (flight

resistance); model parameter estimation (calibration) and model validation.

## 2. DATA CHOICE

Analysis of realized traffic in SM airspace, for years 2003 and 2004, showed that two dominant traffic flows over that airspace exist:

- NW-SE flow (connecting Western Europe with Southern Europe and Mediterranean; e.g. Germany – Turkey, United Kingdom – Greece, etc.) and
- N-SE flow (connecting Northern Europe with Southern Europe and Mediterranean e.g. Sweden – Greece, Denmark – Turkey, etc.).

The flow choice for those two flows is separately modelled. The area of importance for the modelling of traffic over SM was determined and it consists of SM airspace and that of the South-eastern European countries surrounding it (Romania, Croatia, and Italy). Analyzing

realized traffic from EUROCONTROL STATFOR database [2], between specific Origin – Destination (O-D) pairs for the years 2003 and 2004, three main sub-flows were recognized over SM airspace: Northern, Middle and Southern.

For the main flow NW-SE, in the area of interest, according to analysis of traffic flows, three main sub-flows were determined between Austria and Greece (Figure 1):

- Northern sub-flow: Austria – Hungary – Romania – Bulgaria – Greece;
- Middle sub-flow: Austria – Croatia – Serbia and Montenegro – Bulgaria – Greece;
- Southern sub-flow: Austria – Slovenia – Croatia – Italy – Albania – Greece.

In order to get better accuracy of data, data for SM airspace in Middle flow, were subdivided into four main sub-flows (following main airways) over SM according to their share in total traffic (Figure 1).

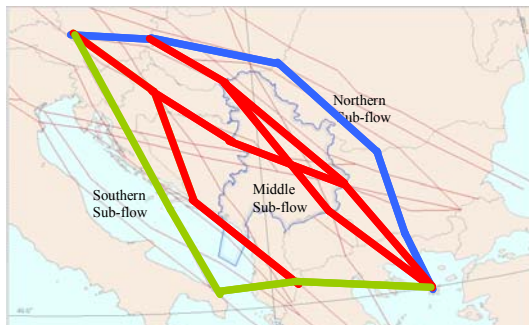


Figure 1. Sub-flows in NW-SE flow

Similarly, for the main flow N-SE, in the area of interest, also three main sub-flows were determined between Slovakia and Greece:

- Northern sub-flow: Slovakia – Hungary – Romania – Bulgaria – Greece;
- Middle sub-flow: Slovakia – Hungary – Serbia and Montenegro – Macedonia – Greece;
- Southern sub-flow: Slovakia – Austria – Slovenia – Croatia – Bosnia and Herzegovina – Italy – Albania – Greece.

After analysis of data between O-D pairs [2], the number of flights to main destinations, over Croatia, Romania, Serbia and Montenegro and Italy, for year 2003 was determined (Tables 1 and 2 respectively for the NW-SE flow and for N-SE flow).

Table 1. Number of flights in NW-SE flow

Destination	NW-SE	Via				
	2003 total	TOTAL	Croatia	Italy	SM	Romania
Turkey	122991	17753	6274	46618	73882	
Greece	115349	45394	67456	53316	4945	
Cyprus	26092	10552	9889	16875	3254	
Egypt	30905	11875	17848	14025	3092	
Bulgaria	19326	2830	216	8454	10819	
<b>TOTAL</b>	<b>314663</b>	<b>88404</b>	<b>101683</b>	<b>139288</b>	<b>95992</b>	

Table 2. Number of flights in N-SE flow

Destination	N-SE	Via				
	2003 total	TOTAL	Croatia	Italy	SM	Romania
Turkey	5457	5	2	48	5359	
Greece	13461	627	476	5203	7750	
Cyprus	2055	5	4	174	1559	
Egypt	1640	4	8	156	1475	
Bulgaria	2143	0	0	14	1476	
<b>TOTAL</b>	<b>24756</b>	<b>641</b>	<b>490</b>	<b>5595</b>	<b>17619</b>	

Taking into account the values for the total number of flights from Table 1 and Table 2, and share of SM sub-flows in the total flow over SM, the number of flights per sub-flow (northern, middle and southern) of NW-SE and N-SE were calculated as well as total number of flights. The values for year 2003 are presented in Table 3. Those values are later used as one of the inputs for the parameter estimation of the flow choice multinomial LOGIT model.

Table 3. Number of flights per sub-flow

Sub flows	Flows			
	2003	NW-SE	N-SE	TOTAL
Northern	95992	17619	<b>113611</b>	
Middle	139288	5595	<b>144883</b>	
Southern	101683	490	<b>102173</b>	
<b>TOTAL</b>	<b>336963</b>	<b>23704</b>	<b>360667</b>	

### 3. VARIABLE CHOICE

Aggregate Multinomial LOGIT model [3] is based on the assumption that the probability of choosing a route or flow  $p_i$  is a function of some explanatory variables, which in our case presents the resistance to the considered flow, i.e. utility of the route to be chosen by an airline. Usually, the probability of choosing a particular flow is higher if the resistance is lower and vice versa. The probability and utility function in the chosen model were calculated using the following equations:

$$p_i = \frac{e^{V_i(x)}}{1 + e^{V_i(x)}} \quad (1)$$

$$\sum_{i=1}^n p_i = 1 \quad (2)$$

$$V_i(x) = a_{0i} + a_1 x_{1i} + a_2 x_{2i} \quad (3)$$

where is:

- $p_i$  probability of choice of flow  $i$ ;
- $V_i(x)$  utility function for flow  $i$ ;
- $x_{1i}, x_{2i}$  resistances for flow  $i$ ;
- $a_{0i}, a_1, a_2$  parameters in equation to be estimated;
- $n$  number of flows.

In reality, the airlines behave differently during the flight planning process, i.e. they choose a particular flow taking into account different criteria. Even more, some airlines on a given O-D pair might choose different routes [4] over time. Usually these criteria are:

- Minimization of the route length;
- Minimization of the route charges;
- Minimization of the total delay;
- Minimization of direct operating costs; etc.

After detailed analysis of traffic and airspace characteristics it was decided that the choice of flow depends on [1]:

1. Unit rate (per country),
2. Average flow length (total and per country) and
3. Average number of available airways (per flow).

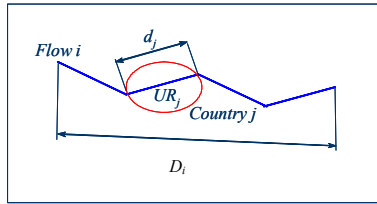
Taking these facts into account, resistances are defined. Here it should be emphasized that variables used to express the resistances are chosen in the way to be predictable (for forecast purposes) and represent the characteristics of a service provider (in some way they present the quality of service – current and future). Resistances were defined in a following way:

**Resistance 1:** combination of flow length and unit rate per country and total flow length (Figure 2):

$$x_{1i} = \sum_{j=1}^{m_i} \frac{d_j \cdot UR_j}{D_i} \quad (4)$$

where is:

- $m_i$  number of countries in flow  $i$ ;
- $d_j$  average length of flow  $i$  in country  $j$ ;
- $D_i$  total length of flow  $i$ ;
- $UR_j$  unite over flight charge in country  $j$ ;
- $n$  number of flows;
- $(i = 1, \dots, n; j = 1, \dots, m_i)$



**Figure 2. Elements of Resistance 1**

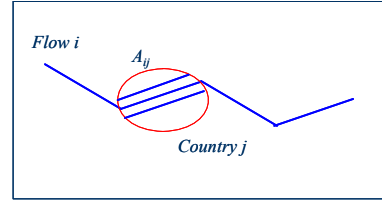
Flow  $i$  traverses the airspaces of  $m_i$  countries. The length of flow in each country  $j$  is  $d_j$  and the unit rate used for calculation of over flight charges  $UR_j$ . From Figure 2 and equation (4) it can easily be seen that the resistance is higher if the unit charge in a country (airspace)  $j$  is higher or if the flow is longer ( $d_j$ ). For each country (airspace)  $j$ , separate resistance values are calculated and normalized, dividing it by the total length of flow  $i$ . Finally, the total resistance in flow  $i$  is calculated summarizing resistance values for each country  $j$  belonging to flow  $i$ .

**Resistance 2:** reciprocal value of average number of airways per country per flow (Figure 3):

$$x_{2i} = \frac{1}{\frac{1}{m_i} \sum_{j=1}^{m_i} A_{ij}} \quad (5)$$

where is:

- $m_i$  number of countries in flow  $i$ ;
- $A_{ij}$  average number of airways in country  $j$  in flow  $i$ ;
- $(i = 1, \dots, n; j = 1, \dots, m_i)$



**Figure 3. Elements of Resistance 2**

The average number of available airways which could be used in flow  $i$  in each country  $j$  is  $A_{ij}$ . Ideally a particular airspace is more attractive to a flight if the number of available airways is higher. In that case the resistance is lower and probability of route choice is higher. If the number of airways is lower, that airspace is less attractive, i.e. resistance is higher and the probability that the airspace will be used for the flight is lower. Resistance values per sub-flows for two main flows (for year 2003) were calculated using the above equations and are presented in Table 4.

**Table 4. Resistance values for main flows (2003)**

Sub flows		NW-SE			N-SE		
		Number of flights	Resistance		Number of flights	Resistance	
			$x_{11}$	$x_{21}$		$x_{11}$	$x_{21}$
	Northern	95992	49.581	0.208	17619	43.513	0.230
	Middle 1	24297	50.791	0.200	2366	44.823	0.286
	Middle 2	26587	51.401	0.250	2588	45.027	0.300
	Middle 3	43751	54.078	0.227	317	44.473	0.333
	Middle 4	44653	54.869	0.273	324	44.202	0.273
	Southern	101683	57.427	0.315	490	52.779	0.600

#### 4. PARAMETERS ESTIMATION

The base year for the parameters value estimation of the model was 2003, driven by the availability of traffic and airspace data. Parameters  $a_{0i}$ ,  $a_1$  and  $a_2$  (see equation 3) were estimated using the maximum likelihood function. Final shape of utility functions for each flow  $i$  are:

Utility function in case of NW-SE flow:

Northern sub-flow:

$$V_1(x) = -10.18659 + 0.19443 x_{11} - 1.79688 x_{21} \quad (6)$$

Middle sub-flow

$$V_2(x) = -12.03389 + 0.19443 x_{12} - 1.79688 x_{22} \quad (7)$$

South sub-flow

$$V_3(x) = -11.43831 + 0.19443 x_{13} - 1.79688 x_{23} \quad (8)$$

Utility function in case of N-SE flow:

Northern sub-flow

$$V_1(x) = -130.32423 + 3.13145 x_{11} - 21.15196 x_{21} \quad (9)$$

Middle sub-flow

$$V_2(x) = -136.65810 + 3.13145 x_{12} - 21.15196 x_{22} \quad (10)$$

South sub-flow

$$V_3(x) = -156.44160 + 3.13145 x_{13} - 21.15196 x_{23} \quad (11)$$

Using these parameter values, the total number of flights per sub-flow were determined and compared with the observed values (realized traffic). The results are shown in Table 5 for NW-SE flow and in Table 6 for N-SE flow.

**Table 5. Estimated vs. observed number of flights for NW-SE flows (2003)**

	NW-SE	Estimated	Observed	Residual	Residual %	Share Estimated	Share Observed
Sub flows	Northern	95992	95992	0	0	0.28	0.28
	Middle 1	25138	24297	841	3.46	0.07	0.07
	Middle 2	25815	26587	-772	-2.90	0.08	0.08
	Middle 3	42803	43751	-948	-2.17	0.13	0.13
	Middle 4	45532	44653	879	1.97	0.14	0.14
	Southern	101683	101683	0	0	0.30	0.30
TOTAL	336963	336963			1	1	

**Table 6. Estimated vs. observed number of flights for N-SE flows (2003)**

	N-SE	Estimated	Observed	Residual	Residual %	Share Estimated	Share Observed
Sub flows	Northern	17619	17619	0	0	0.74	0.74
	Middle 1	2082	2366	-284	-12.00	0.09	0.10
	Middle 2	2816	2588	228	8.81	0.12	0.11
	Middle 3	275	317	-42	-13.25	0.01	0.01
	Middle 4	422	324	98	30.25	0.02	0.02
	Southern	490	490	0	0	0.02	0.02
TOTAL	23704	23704			1	1	

Comparing the estimated number of flights with the observed one (Tables 5 and 6), it was found that model produces satisfactory results.

## 5. MODEL VALIDATION

In order to validate the model, a prediction of the number of flights per sub-flow for year 2004 was made. Validation was made by comparing the estimated number of flights (modelled) with observed, separately for both traffic flows as well as for total traffic. The results for NW-SE flow are shown in Table 7 and for N-SE flow in table 8.

**Table 7. Estimated vs. observed number of transit flights for NW-SE flows (2004)**

	NW-SE	Estimated	Observed	Residual	Residual %	Share Estimated	Share Observed
Sub flows	Northern	93411	102178	-8767	-8.58	0.26	0.28
	Middle 1	25494	23101	2393	10.36	0.07	0.06
	Middle 2	26761	25279	1482	5.86	0.07	0.07
	Middle 3	48333	57044	-8711	-15.27	0.13	0.16
	Middle 4	60534	58221	2313	3.97	0.17	0.16
	Southern	110397	99107	11290	11.39	0.30	0.27
TOTAL	364930	364930			1	1	

**Table 8. Estimated vs. observed number of transit flights for N-SE flows (2004)**

	N-SE	Estimated	Observed	Residual	Residual %	Share Estimated	Share Observed
Sub flows	Northern	7177	19055	-11878	-62.34	0.26	0.69
	Middle 1	5313	3678	1635	44.45	0.19	0.13
	Middle 2	6001	4025	1976	49.09	0.22	0.15
	Middle 3	2129	199	1930	969.85	0.08	0.01
	Middle 4	2632	203	2429	1196.55	0.10	0.01
	Southern	4234	326	3908	1198.77	0.15	0.01
TOTAL	27486	27486			1	1	

From Table 7 it can be seen that in case of NW-SE flow, number of flights over SM airspace is slightly underestimated in total absolute values (sum of Middle 1 to Middle 4 sub-flow – estimated number is 161122, observed number is 163645, the difference is 2523 flights or 1.5% less), while in case of N-SE flow (Table 8) it is overestimated (sum of Middle 1 to Middle 4 sub-flow – the estimated number is 16075, and the observed number is 8105, a difference of 7970 flights or 98% increase). One should be careful when looking at these figures because a large difference in the absolute number of flights between NW-SE and N-SE flow exists (NW-SE: 364930 versus N-SE: 27486 flights). To see how the model estimates the total number of flights in area of interest, Table 9 is produced, containing the figures for

the total number of flights (sum of NW-SE and N-SE flows).

**Table 9. Estimated vs. observed total number of flights in SM airspace (2004)**

	TOTAL	Estimated	Observed	Residual	Residual %	Share Estimated	Share Observed
Sub flows	Northern	100588	121233	-20645	-17.03	0.26	0.31
	Middle 1	30807	26779	4028	15.04	0.08	0.07
	Middle 2	32762	29304	3458	11.80	0.08	0.07
	Middle 3	50462	57243	-6781	-11.85	0.13	0.15
	Middle 4	63166	58424	4742	8.12	0.16	0.15
	Southern	114631	99433	15198	15.28	0.29	0.25
TOTAL	392416	392416			1	1	

From Table 9, it can be seen that the model overestimated the total number of flights in SM airspace in year 2004 by 3.2% relative to the number of observed flights for the same year (sum of Middle 1 to Middle 4, estimated: 177197, observed: 171750). Finally, it was concluded that the model produces satisfying estimates, and that could be used for forecast purposes.

## 6. CONCLUSION

The paper present flow choice model developed for the ATC planning purposes. Model is based on the flow resistances which are modelled as combination of flow length, number of airways and unit rates in the given airspace. Model validation shows satisfactory results. Developed model has good predictive capabilities (on flight distances up to 1000 Nm) important for checking the influence of different market strategies (changes of unit rates and number of airways) on future traffic volume in SM airspace, which should serve as input for airspace and human resources planning. It was planed for further research, to include airline categorization and analysis of their behaviour in order to predict traffic volume more realistically. Special attention shall be given to analysis and comparison of airline behaviour on long hall and short hall flight distances.

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