ABSTRACT: Air Traffic Complexity is discussed and a definition is proposed, as well as desirable properties of a complexity metric. Basic complexity metric structure is further proposed for a particular case of terminal airspace traffic. Two metric models, simple and somewhat more detailed are suggested for consideration. The model outputs are illustrated using a simple generic terminal airspace and traffic demand. Finally, further possible metric improvements are outlined.

KEYWORDS: Air Traffic Control, Air Traffic Management, Traffic Complexity, Terminal Airspace, Modelling

AIR TRAFFIC COMPLEXITY

Complexity of air traffic is here understood as a measure of quantity as well as of quality (characteristics) of interactions between flights, which are to be controlled (managed) by one control subject.

Complexity is further understood as a demand characteristic of a traffic that is to be served by an appropriate supply system, or modified in such a way to make it possible to serve it using an existing supply system.

Primitive complexity is understood to be undisturbed 4D O-D pattern for the considered set of flights. By undisturbed it is understood that each of flights has a 4D trajectory as if it was alone in the considered airspace.

Air Traffic Management is a hierarchical process where activity on each level is meant to produce a less complex output situation, from its input situation, and which is to be handled by the next level.

Complexity may be reduced on strategic, (pre) tactical and operational level. On each of these levels it can have spatial based nature such as air space and airfield system design and/or assignment (air routes, sectors, terminals, runway systems, etc.), but also time based solutions (schedules, slot allocations, flow management, etc).

To be able to evaluate efficiency of possible complexity reduction measures (activities) it is essential to have an appropriate complexity metric.

COMPLEXITY METRIC DESIRABLE PROPERTIES

It is suggested here that it is not appropriate to propose a universal air traffic complexity metric.

The reason is that the nature of managed traffic, and thus the complexity, differs significantly between e.g. “free flight” type traffic over a large area such as USA or European airspace and say traffic on the two intersecting runways under saturation.

Interactions between flights are different in the above mentioned situations and the proposed metrics should reflect that fact.

On one hand a metric should take into account all relevant traffic characteristics, but on the other hand these should be only the significant ones.

Another question is how large the set of considered flights should be. It can vary from very large e.g. when a strategic or a (pre) tactical problem such as sectorization (sector design and/or grouping) is considered, to rather small when an operational problem like sequencing at the final approach is solved.
Connected to this is also the problem of a time horizon for which complexity is measured. One extreme is to have a snapshot approach, while some type of a look-ahead approach seems more appealing. The second approach takes into account all that is known about flight “intentions” during the (relevant) future time (window) period.

If the metric is supposed to reflect the usually very dynamic nature of complexity it should probably “look” into the sliding time window presenting the traffic situation in the considered space.

**TERMINAL TRAFFIC CHARACTERISTICS**

Terminal traffic usually has the following characteristics:

- Traffic arrives from several destinations and departures are to similar destinations;
- Trajectories converge to the landing runway(s) and diverge from departure runway(s);
- The fleet can have such mixture that the whole range of aircraft sizes and velocities and consequently separations are present.

Based on the primitive traffic complexity, and usually taking into account existing and planned airfield infrastructure in the region, the design of the terminal airspace is decided on. It includes the geometry of the airspace and often the flow pattern structure.

Arrival flows are physically separated from departure flows whenever possible, so that if the prescribed routes are followed there is no interference between these two types of traffic.

**TERMINAL AIRSPACE GENERIC CASE**

To propose a metric for terminal air traffic complexity and to illustrate it, a generic case of terminal airspace is used here (Figure 1).

![Figure 1. Terminal Airspace Generic Case](image)

The case takes into account arrivals only, assuming that the departures are separated on different (standard) routes.

The terminal has only two entry points: EP₁ and EP₂. There is only one runway threshold in the terminal, which implies one final approach fix (FAF).

There are no missed approaches. All arrivals go directly from entry point to the final fix, i.e. follow EPₙ - FAF path. The above-described case will be used in further discussion, as the first step towards developing an appropriate metric.
BASIC COMPLEXITY METRIC STRUCTURE

Following the discussion in part 2 the question that arises is: which are the main components of the terminal airspace traffic complexity?

Traffic density is generally considered as one of the key components of the complexity. Obviously the increasing number of arriving, converging flights implies an increased number of expected interactions.

The following question is: which are the interactions that appear between arriving flights in a terminal?

One of the interactions is the case when the two aircraft follow the same route (between EPₚ and FAF), the trailing being faster than the leading one; the separation between the two aircraft decreases, producing catching-up case.

Another interaction is the case when two aircraft approach FAF from different entry points. This case can be understood as the two aircraft flows merging into one. Additionally the case of each aircraft approaching FAF can be understood as that aircraft merging into the other EPₚ - FAF - THR flow.

In regular terminal arrival traffic management there does not seem to be any more significant interaction cases in a regular situations (no missed approaches).

One may conclude that the basic structure of proposed complexity metric would be:

\[ C = C' + C'' + C''' \]

where:

\[ C' \] - is traffic density complexity component;  
\[ C'' \] - is catching-up complexity component;  
\[ C''' \] - is merging complexity component.

Complexity is dynamic, and consequently follows that:

\[ C(t) = C'(t) + C''(t) + C'''(t) \]

The density component, \( C'(t) \) can be considered to be relevant as value observed at the time \( t \) which changes only at the discrete moments when new flights enter the system (at EPₚ) or flights leave the system (at FAF).

To find out the value for the catching-up and merging components it is proposed here to examine all such cases for the flight population in the system at time \( t \). This implies a look-ahead type procedure for the time horizon equal to the time necessary for the last entry flight to reach FAF.

The evaluation procedure should be repeated (metric value “refreshed”) every time the new entry appears at any EPₚ. This implies that density increases and that the catching-up and merging components might increase as well. The value of the metric should be re-evaluated also when any of flights leave via FAF. This implies density decrease as well as the possible decrease of the other two components.

Any missed approach, with the flight coming back into the arrival stream will increase traffic complexity. This component will be studied separately and possibly later included in the model.
Model 1 – Simple Statistics

This model proposes a simple statistics about the three above described complexity components.

The first component, density, could be evaluated comparing number of flights in the terminal with the throughput of FAF during the time period necessary for the last entry to travel the distance from its entry point to FAF.

This is actually a measure of saturation. One possible assumption is that the number of entries will not be higher than the throughput of the “exit” (FAF) in the coming period, i.e. that the system will not be overloaded. It follows:

\[ C'(t) = \frac{N(t)}{Q}, \quad 0 \leq C'(t) \leq 1 \]

where:

- \( N(t) \) – number of flights in the terminal at time \( t \);
- \( Q \) – estimated throughput of FAF.

However one can argue that this is not necessarily realistic as there are (or at least used to be) such overload periods solved using holding queues inside terminals. We will not try to include this case in this (initial) stage of the research.

The second component, catching-up, can be defined as the faction of flights that are catching up with the leading flights at the same EP\_p - FAF route. This component can be

\[ 0 \leq C''(t) \leq \frac{N(t) - 1}{N(t)} < 1 \]

The minimum value is in the case where are no any trailing flights faster than corresponding leading flights. The maximum value is achieved in the case where the velocity of each next entering flight increases, and the ratio can be close to one.

The third component, merging, can be defined as the faction of flights from the flow with lower intensity i.e. smaller number of flights into the other flow with more flights. It follows that

\[ C'''(t) = \frac{2 \cdot N(t)}{N_1(t) + N_2(t)} \]

for \( N_1(t) < N_2(t) \), where:

- \( N_1(t) \) and \( N_2(t) \) are numbers of flights on routes EP\_1 - FAF and EP\_2 – FAF respectively, at time \( t \).

This third component could have the following values:

\[ 0 \leq C'''(t) < 1 \]

The case when all flights are on the same route, i.e. there is no merging, will have for result value 0, while the even split of flights on two routes will produce the value 1.

In the simplest case the three components can add up producing

\[ 0 \leq C(t) < 3 \]
The authors do not necessarily promote the idea of equal weights for all components so that more general model could be

\[ C(t) = w' \cdot C'(t) + w'' \cdot C''(t) + w''' \cdot C'''(t) \]

where each component could be assigned particular weight \( w' \). At the present research stage not enough information is available to discuss the possible values of weights.

Even more general the form of the model could be:

\[ C(t) = f'(C'(t)) + f''[C''(t)] + f'''[C'''(t)] \]

where \( f', f'', f''' \) could be non-linear functions, e.g. penalising high values of any of the three components.

**Model 2 – Critical Interactions**

This model treats all three components in some more details. It tries to define the relationship between flights on the same route or on different routes.

Instead of simple density it considers how the flights are distributed on the EP_p · FAF routes.

In evaluating the second and the third component not all the catching up and merging interactions are considered and have same contribution, such as in Model 1, but only the ones that would result in violation of the minimum separation rules along the route or at FAF if no action is undertaken by the controller.

The threshold throughput \( Q \) is again used as the reference traffic volume level. Reciprocal value of \( Q \) presents the average inter-arrival time between two aircraft on the final approach (FAF). This value dictates the average inter-arrival time value on routes. Reciprocal value of this time presents the routes throughput \( Q_1 \) and \( Q_2 \) under the constrain:

\[ Q = Q_1 + Q_2 \]

Density of aircraft per distance unit \( G \) (for each route) is calculated based on the average aircraft speed \( v_{avg} \):

\[ G_1 = \frac{Q_1}{v_{avg}}, \quad G_2 = \frac{Q_2}{v_{avg}} \]

Multiplying the given density by the route length \( d \) we will produce the value for maximum number \( N_p^{\text{max}} \) of aircraft instantaneously present on route \( p \):

\[ N_1^{\text{max}} = G_1 \cdot d_1, \quad N_2^{\text{max}} = G_2 \cdot d_2 \]

Now, the values of the complexity component can be computed. The value of each component in the basic model form will be between 0 and 1, similar to Model 1. The form of the components will in general allow for more entry points and routes but will here be limited to the generic case, so that \( p \), number of routes, will be taking values 1 and 2.

The traffic density component is calculated in the following way:

\[ C'(t) = \frac{1}{2} \cdot B(t) \cdot \left[ \sum_p N_p(t) \right] \]

were:

\( B_p(t) \) - number of routes in use.

Catching-up component will be calculated in the following way:
\[ C^*(t) = \frac{1}{2} \cdot B(t) \cdot \left[ \sum_p y_p(t) \cdot \sum_s \left( \frac{\Delta T - T_s}{\Delta T} \right) \cdot \frac{\Delta T}{\Delta T} \right] \]

were:

- \( A_p(t) \) - number of lead-trail flight pairs on the route \( p \);
- \( y_p(t) \) - binary variable with value equal to 1 when catch-ups exist, 0 otherwise;
- \( \Delta T \) - minimum time separation between flights;
- \( T_s \) - estimated time separation between flights pair \( s \) at the FAF (\( s = 1 \) to \( A_p(t) \)).

The merging complexity component is calculated in the following way:

\[ C^m(t) = \frac{1}{2} \cdot B(t) \cdot \left[ \sum r z(t) \cdot \sum \left( \frac{\Delta T_r - T_r}{\Delta T_r} \right) \cdot \frac{\Delta T_r}{\Delta T_r} \right] \]

were:

- \( D(t) \) - number of flight pairs with merging interaction (consecutive arrivals at FAF from different routes);
- \( z(t) \) - binary variable with value equal to 1 in case when merging conflict exist, 0 otherwise;
- \( \Delta T_r \) - minimum time separation in FAF;
- \( T_r \) - estimated time separation between aircraft pair \( r \) at the FAF (\( r = 1 \) to \( D(t) \)).

The discussion about different weights and/or possible non-linear nature of transformation functions for Model 1 might be applied for Model 2 as well.

**NUMERICAL EXAMPLE**

A simple numerical example is produced to illustrate the outputs of the two proposed complexity metric models. The simplest versions of the models are used, namely with no weights applied, so that the possible range of obtained values is 0 to 3. Terminal configuration is as presented in Figure 1. The distance between entry points \( EP_1 \) and \( EP_2 \) and the FAF are equal to each other and 60 Nm long. A fleet consisting of two types of aircraft with velocities of 210 and 240 kts enters the terminal.

Maximum threshold and consequently FAF throughput is 15 flights per hour. This results in an average inter-arrival time at FAF of 2 minutes. The entries at the two points \( EP_1 \) and \( EP_2 \) are generated using Monte Carlo simulation with equal probability of entries.

Figure 2 shows the number of flights present on each of two routes during the hour that is used for illustration. Figure 3 shows the cumulative of the graphs of aircraft presence on the routes, i.e. the number present in the system, between entry points and FAF. Figure 4 shows the values of complexity (index) calculated using Model 1, for each component separately as well cumulative value. Figure 5 shows similar results from Model 2.

By comparing complexity of separate components it may be noticed that the two models present density component in a similar manner. However this is not the case when catching up and merging components are presented, as expected when taking into account the intended nature of the two models.
It can be noticed that Model 2 shows that there is barely any catching up complexity problems and merging complexity appears only during second half of the analysed hour.

When the compound complexity is observed it can be noticed that the two models produce much more similar results. Roughly speaking both models show that the complexity during the second third of the hour doubles compared to the first third hour period, and then again drops in the last third on the level which is about 50% higher than in the first third hour.

In order to enable complexity comparison, as shown in Figure 4d and Figure 5d, it is proposed that the cumulative duration of the compared indices be observed. The rationale behind this is that the cumulative duration of the value of an index might be more relevant then the time when it occurred. This might be valid for many phenomena in traffic analysis (Tosic, 1999.).

![Figure 2. Number of flights per route](image1)

![Figure 3. Number of flights in the system](image2)
Figure 6 shows the relation between indices computed using Model 1 and Model 2. Indices cumulative duration is here presented in the descending order. It can be seen that Model 1 gives systematically higher index values which is to be expected as it takes into account all the “candidate” interactions whether they lead to an incident or not. However, it can be concluded that both models produce similar cumulative duration patterns.
When a metric is proposed it is very important to know how well it reflects changes in some of the variables or parameters of the observed system.

Figure 7 shows the results of an experiment using Model 2 when traffic volume is changed. It is obvious that the proposed metric – index, reflects very well the change of traffic volume.

Figure 8 shows similar features of Model 2 product (index), i.e. sensitivity to a change of terminal airspace geometry, namely number of arrival patterns.
CONCLUSIONS

Two models for terminal airspace complexity metric are proposed. The intention is to provoke discussion in the research community concerned with quantifying ATM performance improvement. Namely many more efforts are made in the area of “optimisation” or “decision support” than in the field of measuring the state of the managed system.

Traffic complexity is systematically treated in an effort to reduce it through different management actions and so facilitate its handling by ATC operations.

To see how successful possible complexity reduction actions are one requires an appropriate metric. This metric should be relatively easy to evaluate (compute), reflecting relevant variables and parameters of the process including sensitivity to the changes of these variables and parameters.

The same metric can be used to analyse the difficulty that complexity imposes on the operator as well as on the system’s performance.

Further development in the area of terminal airspace traffic complexity metric is suggested. It should include interviewing experienced operators with the aim of improving the form of the function connecting the complexity components. It should also take into account possible impact of the complexity components reflecting irregular situations such as missed approaches coming back into the arrival stream.

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