TERMINAL AIRSPACE TRAFFIC COMPLEXITY

Fedja Netjasov, University of Belgrade, Faculty of Transport and Traffic Engineering
Belgrade, Serbia and Montenegro

Introduction

Is the traffic situation complex or not? Most researchers are trying to answer this question in a sense to determine the influence on the traffic controller. Shaefer [1] defines traffic complexity as a “measure of the difficulty that a particular traffic situation will present to an air traffic controller”. Controller workload is connected with complexity and depends on the following factors: “the geometrical nature of air traffic, operational procedures and practices used to handle the traffic and the characteristics and behavior of individual controllers”. Delahaye and Puechmorel [2] try ‘to synthesize a traffic complexity indicator in order to better quantify the congestion in an air sector’. That indicator is based on “complexity of distribution of traffic in airspace”. A metric, defined in such a way, could be used in many applications in the ATM area such as: “balancing of sector congestion during sectorization, traffic assignment with the aim to reduce congestion, design of a new air network, for concept of dynamic sectoring, for qualification and quantification of Air Traffic Service providers performances”. Laudeman [3], [4] defines a concept called “dynamic density” which includes traffic density (number of aircraft in airspace) and traffic complexity (measure of traffic complexity in specific airspace). This concept should serve as a metric of controller workload. Pawlak et all, [5], [6], [7], [8] address ATC Complexity and they define the framework for evaluation of complexity. Numerous initial factors influencing traffic complexity are shown.

In addition to the mentioned authors other researchers have tried to define and measure complexity in some way, such as [9] to [15]. But, we conclude that most of them deal with en-route complexity and attempt to find a connection between complexity and air traffic controller workload. However, we believe that complexity could also have an influence on some other aspects such as environment, air carrier delays and operational costs, quality of passenger service etc.

In this paper we focus on terminal airspace (TMA) as a transitional airspace between the airport and en-route sectors. Why?

Because of the huge concentration of arrival paths converging on an airport and departure paths diverging from an airport. Also, because there exist three main ways of flying through the TMA (following RNAV and NON RNAV SID’s and STAR’s and vectoring by traffic controllers). Aircraft traverse the TMA at a broad spectrum of speeds. During arrival, speed is either constant (for all aircraft due to traffic) or is different (for different types of aircraft) and mostly lower or equal to 250kt. Speed decreases through the TMA and can have the value of 140 to 150kt in the touch-down zone on runway. Similarly, during departure, the speed of aircraft increases from 130 to 150kt at take-off to over 250kt at the TMA exit point. Bearing all the mentioned characteristics in mind and also taking into account aircraft fleet mix and weather changes, one can see how complex a system the TMA is.

In this paper, The aim was to define generic factors influencing traffic complexity in terminal airspace. For that purpose a metric is proposed, called the Index of Complexity consisting of a static and dynamic part. We believe that the proposed metric could have an application for TMA traffic management at the operational level as well as for traffic planning on a tactical and strategic level, e.g. what influence could construction of a new runway or implementation of a new approach or departure trajectory have on traffic in the TMA. Also, we think that information about complexity could be used for prediction of controller workload, flight delays, airline operating costs, noise levels, emission of pollutants, etc.

Measure of complexity - Concept

Thinking about the problem of measuring complexity and considering the available literature from this field, it was concluded that insufficient consideration has been given to the measurement of complexity, independent of the area of its influence (e.g. independent of workload, ecological influence
and similar). The idea, which arises from this observation was to define a measure which will be “objective”, i.e. that will not take into account the possible influences of complexity, but will simply consider a section of airspace and the traffic within it. In that way, a measure, which could serve as a variable for determination of system influence on other elements of the environment, is attainable. It should be born in mind that between complexity and some elements of the environment a “reason /consequence” relation could exist.

In this paper it is assumed that complexity presents a measure of quantity as well as quality of the interactions between the aircraft which are to be controlled (managed) by one air traffic controller. The metric, called Index of Complexity, is proposed to serve as a TMA system performance measure [16].

The modeling of the complexity measure for traffic in the TMA, is based on an assumption that consists of two basic parts: static and dynamic. The static part \(C_s\) includes the terminal airspace geometry – the shape and dimensions of TMA, the number of airports, number and length of approach and departure trajectories, number of entry and exit points, etc. The dynamic part depends of traffic demand characteristics (distribution of arrival and departure traffic, aircraft mix, etc.) and distribution of traffic inside the TMA (distribution of traffic on trajectories, separation rules between aircraft, etc.).

The dynamic part is divided into arrival \((C^{d}_{ARR})\) and departure \((C^{d}_{DEP})\) parts. It was assumed that the linear sum of all the mentioned parts presents the Index of Complexity \(C\):

\[
C = C_s + C^{d}_{DEP} + C^{d}_{ARR}
\]

Concept assumptions

Analyzing the TMA we define some assumptions. The basic assumption is that the subject of analysis is a real, existing TMA with all characteristics: number of entry and exit points, number of runways in use for departure and arrival, shape, dimensions, etc.

Considering the traffic in the TMA, we conclude that besides flying through the TMA, one aircraft could also be in position to possibly overtake (cach-up) the preceding aircraft on the same trajectory (overtake conflict) or to be insufficiently separated at the point were trajectories are merged, from an aircraft coming from another trajectory (merging conflict). Also it is assumed that the possibility of traffic demand exceeding the capacity of a trajectory, exists.

According to those assumptions the Index of Complexity in case of arrivals is defined as the sum of four indices:

- \(C'_{ARR}\) - complexity induced by arriving traffic;
- \(C''_{ARR}\) - complexity induced by overtakes (caching-up situations);
- \(C'''_{ARR}\) - complexity induced by merging conflicts;
- \(C''''_{ARR}\) - complexity induced by demand exceeding capacity of a trajectory.

Similarly, in case of departures the Index of Complexity is defined as the sum of two indices:

- \(C'_{DEP}\) - complexity induced by departing traffic;
- \(C''_{DEP}\) - complexity induced by overtakes (caching-up situations).

The main assumptions in the model are:

- The subject of analysis is a TMA which contains one single runway airport;
- \(m\) is finite number of entry points in TMA;
- \(k\) is finite number of exit points from TMA;
- \(p\) is number of arriving trajectories inside TMA \((p \in P_{ARR} \text{ finite set of arriving trajectories and } p = 1 \text{ to } m)\);
- \(r\) is number of departing trajectories \((r \in R_{DEP} \text{ finite set of departing trajectories and } r = 1 \text{ to } k)\);
- the number of arriving trajectories connecting one entry point with the threshold could be one (similar for departing trajectories connecting threshold and an exit point);
- arrival and departure trajectory are not crossing then vertically separated;
- threshold throughput (number of arriving aircraft per hour) is known in advance;
- two categories of aircraft are considered (slow and fast) and their speeds are known;
- value of indices of complexity are absolute (not relative);
- predefined flight trajectories are used (concept does not imply the existence of vectoring).

The basic idea of the model is that system state and complexity are changed whenever an aircraft enters or leaves any trajectory.
Index of static complexity

The Index of static (structural) complexity ($C_s$) can be defined in the following way:

$$C_s = \frac{m \cdot \sum_{i=1}^{m} d_i^{ARR} + k \cdot \sum_{j=1}^{k} d_j^{DEP}}{P(Nm^2)} \cdot 1Nm$$

were:

- $d_i^{ARR}$ - length of arrival trajectories $i$;
- $d_j^{DEP}$ - length of departure trajectories $j$;
- $1 Nm^2$ - serves only for cutting unit of measures, i.e. to obtain no-dimensional number;
- $P$ - size of TMA (surface).

From this equation it can be concluded that increasing number of trajectories in TMA causes increase of complexity, and on the other hand, increasing of the TMA size, without increasing the number of trajectories, causes complexity decrease. The Index of static complexity, defined in such a way, offers the possibility of comparing the complexities of different airport TMAs (e.g. TMA Zurich vs. TMA Munich).

Index of dynamic complexity for arrivals

Runway threshold throughput $Q_{ARR}$ (aircraft per hour) is known in advance. Reciprocal value of $Q_{ARR}$ presents the average inter-arrival time between two aircraft on final approach. This value is dictating the average value on trajectories. Reciprocal value of aircraft on final approach. This value is dictating the presents the average inter-arrival time between two arrivals.

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\[ N_p^{\text{max}} = G_p \cdot d_p \]

Now, Index of complexity induced by arriving traffic ($C_{ARR}$) is calculated in the following way:

\[ C_{ARR}(t) = B_p(t) \left( \frac{\sum_{p=\text{ARR}}^{N_p^{\text{max}}} N_p(t)}{\sum_{p=\text{ARR}}^{N_p^{\text{max}}}} \right) \]

were:

- \( B_p(t) \) – is the number of arrival trajectories in use at a moment $t$, i.e. on which traffic exists;
- \( N_p(t) \) – is the number of aircraft present on arrival trajectory $p$ at moment $t$.

If it happens that $N_p(t) = N_p^{\text{max}}$ then $C_{ARR}$ will have a maximum value equal to the number of trajectories in use $B(t)$ at moment $t$. The number of aircraft on trajectory $p$ at any moment of time $t$ belong to the range: $0 \leq N_p(t) \leq N_p^{\text{max}}$. Number $N_p(t)$ could be larger than $N_p^{\text{max}}$ which will automatically lead to the appearance of a holding procedure while condition $N_p(t) \leq N_p^{\text{max}}$ is not met again. Such a situation will produce the complexity induced by demand exceeding trajectory capacity.

Value of index of complexity induced by overtakes ($C''_{ARR}$) will be calculated in the following way:

\[ C''_{ARR}(t) = B_p(t) \left( \frac{\sum_{p=\text{ARR}}^{N_p^{\text{max}}} \Delta T \cdot T_p}{\sum_{p=\text{ARR}}^{N_p^{\text{max}}} \Delta T} \right) \]

were:

- \( A_p(t) \) – is the number of aircraft pairs on trajectory $p$ between whom a possible caching-up conflict (overtake) exists;
- \( y_p(t) \) – is the binary variable with value equal to 1 when the possibility for caching-up exists, 0 otherwise;
- \( \Delta T \) - standard temporal separation between aircraft pair on trajectory $p$;
- \( T_p \) - estimated temporal separation between aircraft pair at the exit point of trajectory $p$.

When an aircraft enters trajectory $p$ and if there is an aircraft in front of it which is slower, then estimation of temporal separation between those two aircraft at exit point ($T_p$) is made. If the estimated value is lower than standard separation, caching-up could happen. If the difference ($\Delta T - T_p$) is higher, then
the intensity of possible caching-up conflict is higher and also the value of the Index of complexity.

The Index of Complexity induced by overtakes exists from the moment when a faster aircraft enters the trajectory $p$ until the moment when the slower aircraft exits the same trajectory.

The third index of complexity, induced by merging conflict ($C'''_{\text{ARR}}$) is calculated in the following way:

$$C'''_{\text{ARR}}(t) = B_p(t) \cdot \frac{z(t) \sum_{p=1}^{\text{max}} \left( \Delta T_f - T_s \right)}{\sum_{p'=\text{ARR}} N_{\text{max}}^{p'}}$$

were:

- $D(t)$ – the number of aircraft pairs from different trajectories, between whom possible merging conflicts exist;
- $z(t)$ – the binary variable with a value equal to 1 in case when possibility for merging conflict exists, 0 otherwise;
- $\Delta T_f$ - the standard temporal separation between an aircraft pair from different trajectories, in final approach;
- $T_s$ – the estimated temporal separation between aircraft pair from different trajectories, at a point where trajectories are merged.

Comparison of exit time from trajectories, at the merging point, for a pair of aircraft on different trajectories is made. If the temporal separation between those aircraft $T_s$ (in a situation when one of them reaches the merging point) is less than standard temporal separation, then we assume that merging conflict could occur. The difference $(\Delta T_f - T_s)$ indicates the intensity of possible merging conflict, i.e. if time difference is higher than the intensity of possible conflict then the value of the Index of complexity is higher.

The index of complexity induced by merging conflict exists from the moment when one aircraft enters a trajectory until the moment when another aircraft exits the other trajectory, which is merged with the previous one.

The model also contains an index of complexity induced by traffic overloading ($C'''_{\text{ARR}}$). Namely, in a situation when traffic demand is higher than available trajectory capacity (in this case $N_p^{\text{max}}$) then additional complexity is generated. The value of this index is calculated in the following way:

$$C'''_{\text{ARR}}(t) = \sum_{p=1}^{\text{max}} g_p(t) \cdot \left[ \frac{N_p(t) - N_p^{\text{max}}}{\sum_{p'=\text{ARR}} N_{\text{max}}^{p'}} \right]$$

were:

- $g_p(t)$ – is a binary variable with value equal to 1 in cases when overloading exists, 0 otherwise.

A greater difference $(N_p(t) - N_p^{\text{max}})$ produces a higher value of index. Such an index exists from the moment when the aircraft overloading the trajectory $p$ enters into it, until some previous aircraft exits the same trajectory.

### Index of dynamic complexity for departures

Departures are under greater control by air traffic controllers which as a result give us the assumption that the “overloading” of a trajectory is impossible. On the other hand, there is no chance that phenomena similar to merging (as in the case of arrivals) can occur because the singular flow after take-off, which is under air traffic controller competence, is decomposed into smaller flows (departure trajectories).

At the moment when an aircraft, after take-off, enters a trajectory $r$, the number of aircraft present on trajectories increases and the system state changes, as well as the value of complexity. The relation between the actual number of aircraft $N_r(t)$ and the maximum available number $N_r^{\text{max}}$, represents the value of the Index of complexity. A higher number of aircraft causes greater complexity. It is also important how many trajectories are in use at moment $t$. Each over how many trajectories traffic is distributed. It is believed that traffic on a higher number of trajectories generates greater complexity. The value of complexity will be calculated in following way:

$$C''_{\text{DEP}}(t) = B_r(t) \cdot \left[ \frac{\sum_{r=\text{DEP}} N_r(t)}{\sum_{r=\text{DEP}} N_{\text{max}}^{r}} \right]$$

were:

- $B_r(t)$ – is the number of departure trajectories in use at moment $t$, i.e. on which traffic exists;
- $N_r(t)$ – is the number of aircraft present on departure trajectory $r$ at moment $t$. 
Potential overtake will generate additional complexity which will be presented by Index $C''_{DEP}(t)$. The Index value will be calculated in the following way:

$$C''_{DEP}(t) = B_r(t) \cdot \left[ \sum_{r \in R_{dep}} y_r(t) \cdot \sum_{r = 0}^{A_r} \left( \frac{\Delta T - T_r}{\Delta T} \right) \right] \sum_{r \in R_{dep}} N_{max}$$

were:

- $A_r(t)$ – is the number of aircraft pairs on trajectory $r$ between whom possible caching-up conflict (overtake) exists;
- $y_r(t)$ – is the binary variable with a value equal to 1 in cases when the possibility for caching-up exists, 0 otherwise;
- $\Delta T$ - is the standard temporal separation between aircraft pairs on trajectory $r$;
- $T_r$ – is the estimated temporal separation between aircraft pairs at the exit point of trajectory $r$.

From the expression it can be seen that a lower value of $T_r$, produces higher intensity of potential overtake, hence the value of complexity increases, and vice versa, a higher value of $T_r$ generates lower complexity. From duration point of view, the overtake is considered to have begin at the moment when a faster aircraft enters a trajectory $r$ and to have ended when the initial, slower aircraft leaves the departure trajectory.

**Numerical example**

A simple numerical example, considering only arrivals, has been produced to illustrate the outputs from the model. A generic TMA configuration is used containing two arrival trajectories (Figure 1). The distance between entry points (EP 1 and EP 2) and the final approach fix (FAF) are equal to each other and 60Nm long. A fleet consisting of two types of aircraft with velocities of 210 and 240kt enters the TMA.

Let assume that maximum threshold and consequently FAF throughput is 15 flights per hour. This fact results in an average inter-arrival time at FAF of 2 minutes. Aircraft entry times at two points (EP 1 and EP 2) are generated using Monte Carlo simulation with equal probability of entries.

Figure 2 shows the cumulative number of aircraft present on both trajectories, i.e. between entry points and FAF. Figure 3 shows the values of the resulting complexity (index). Comparing figures 2 and 3 it can be seen that the same number of aircraft does not produce, at any time, the same value of Index of Complexity. This fact proves the basic assumptions and complexity definition, i.e. that complexity depends of quantity as well as quality of interactions.

In order to enable complexity comparison, it is proposed that the cumulative duration of the compared indices be observed. The rationale behind this is that the cumulative duration of some index value, might be more relevant than the time when it occurred [17].

When a metric is proposed it is very important to know how well it reflects changes in some variables or parameters of the observed system. Traffic volume (threshold throughput) and number of arrival trajectories are chosen to examine the sensitivity of the model. It was assumed that growth...
of both variables will produce growth of Index of Complexity.

Figure 4. shows the cumulative duration of the Index value, in case when traffic volume is changed. It was shown that an increase of traffic volume produces an increase of Index of complexity as well as the duration of the complex situation.

This fact proves the basic assumption. Namely, increase of the traffic volume by 33% and 66% will produce the difference in value range (18 aircraft – complexity value between 0 and 5; 24 aircraft – 1.5 to 5.5 and 30 aircraft – 3 to 6.5). Duration of complex situation is also increased so, e.g. a situation with value $C = 4$: in a case with 18 aircraft last ~5% of the observed hour, for 24 aircraft ~20% and for 30 aircraft ~70%.

**Figure 4. Sensitivity to Traffic Volume Changes**

Figure 5. shows sensitivity to a change of terminal airspace geometry, namely the number of arrival trajectories. It can be seen that with increases in the number of trajectories, the index of complexity also increases as well as duration of complex situation. This feature seems to be reasonable like previous one (increase of traffic volume). Changes in the number of trajectories produce the difference in value range (2 trajectories – complexity value between 1.5 and 5.5; 3 trajectories – 2 to 7.5 and 4 trajectories – 3.5 to 9.5). The duration of the complex situation is also increased so, e.g. situation with value $C = 4$: in case with 2 trajectories last ~25% of the observed hour, for 3 trajectories ~75% and for 4 trajectories ~95%.

**Figure 5. Sensitivity to Terminal Airspace Geometry Changes**

**Conclusion**

Starting with idea of quantifying terminal airspace complexity, this research resulted, firstly with the concept of complexity and then with a model of the Index of traffic complexity. The Index of traffic complexity presents a measure (metric) for estimation of the state of a given system, which is in nature very specific. Namely, terminal airspace represents a system which serves as a transition between airport and ATC sectors. This airspace contains a network of trajectories converging on the airport (in case of arrivals) and diverging from the airport (in case of departures). Aircraft of different types and speed ranges fly on this network.

The main result of this research is the development of a model for the Index of complexity. This model could be used for evaluation of current and novel organizational solutions for a TMA containing a single runway airport, or for estimation of effects of new arrival and departure trajectories implementation, on traffic in a TMA.

Complexity is defined as a measure of quantity as well as of quality (characteristics) of interactions between aircraft flying through the TMA. The model is based on the assumption that complexity contains a static and dynamic component. The dynamic component is concerned separately with arrival and departure. In case of arrival, four components influence complexity: existence of traffic, possibility of catching-up (overtake) of aircraft flying on the same trajectory, possibility of conflict occurrence at trajectory merging point, and possibility of overloading trajectory which will lead to aircraft holding. The case of departing traffic is a bit simpler and contains two components: traffic existence and possibility of aircraft catching-up (overtake).

The mentioned components are assumed to be regular situations, i.e. interactions, between aircraft flying through the TMA. Irregular situations, such as, missed approaches, currently aren’t considered in this model.

Finally, the problems considered in this research, are just the beginning of research in the field of terminal airspace traffic complexity. There is a great deal of space for further research as well as open questions such as: consideration of irregular situations such as missed approaches; consideration of influences of the meteorological situation on traffic; analysis of traffic complexity for airports with multiple runways; usage of weight factors and non-linear summation of complexity components; consideration of heterogeneous aircraft fleet, etc.
References


Biography

Fedja Netjasov is Research and Teaching Assistant at the Division of Airports and Air Traffic Safety, Faculty of Transport and Traffic Engineering, University of Belgrade. He obtained his B.S. (Dipl. ing.) from the Faculty in 1999. He received M.S. from the same Faculty in July 2003. Major fields of interest are: ATM, Airports, Modeling, Transport and Traffic System Analysis.