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# Airspace daily operational sectorization by fuzzy logic

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**Abstract**

From the point of view of Air Traffic Control the airspace of a country or region are divided into sectors. Each sector is served by a team of controllers. Depending on traffic characteristics (volume, scenario, aircraft types, etc.) several sectors can be merged into one sector or work independently. When the traffic characteristics are known in advance (planned or forecasted) the problem that arises is to determine the number of open sectors during a given time period (day, week, etc.) so that the traffic requesting service can be served with an acceptable work load for the controllers. In this paper, a decision support tool based on fuzzy logic is proposed to solve the above-mentioned problem. The proposed decision support tool is illustrated by a numerical example from real life. © 2000 Published by Elsevier Science B.V. All rights reserved.

*Keywords:* Air Traffic Control; Sectorization; Fuzzy sets; Approximate reasoning

## 1. Introduction

The regulation and flight control of an airspace is provided by Air Traffic Control (ATC). Each country is responsible for providing ATC for its airspace either as a separate ATC or part of the ATC which is responsible for the airspace of two or more countries.

The primary duties of ATC are to provide safe, regular and expeditive air traffic in the airspace for which it is responsible. To carry out these duties the airspace is divided into sectors which are defined by horizontal and vertical borders. The regulation and traffic control of each sector is performed by a controller or team of controllers. Each sector has a certain capacity depending on several factors:

- ATC system (non-radar, radar).
- Human factors – controllers' experience.
- Traffic characteristics – scenario (overflights, climbing, descending, military activity, etc.).
- Airspace structure (the number of airways, one- or two-way airways, airway crossings, etc.).
- Technical equipment of aircrafts and ATC system, etc.

The sector capacity can be defined in several ways:

- as the maximum number of aircraft which can be served during a certain time period $\Delta t$, or
- as the maximum number of simultaneously present aircraft in a sector, or
- as the maximum number of aircraft which can be served during a certain time period $\Delta t$, under the condition that the number of simultaneously present aircraft is not greater than a given number during the considered $\Delta t$.

Depending on the volume and characteristics of the expected traffic and available number of controllers

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in a shift, a dispatcher (an expert controller) in the Area Control Center (ACC) makes a decision about the daily airspace operational division into sectors. Each of the daily sectorizations must cover the whole airspace the ACC concerned is responsible for. Namely, depending on the above-mentioned factors, from the set of sectors defined in advance, the dispatcher decides which sectors to open (during the shift) so that the opened sectors provide control of the whole airspace. Henceforth, the dispatcher can merge a few sectors into one or can divide an opened sector (merging the airspace of a few sectors) into separate sectors.

When opening new sectors, the sectors are put into work sequentially (one by one) according to a predefined schedule. A predefined schedule to open sector means that for a given number of open sectors it is known in advance which sectors will be opened. The procedures applied to the closing of sectors is similar. Thus, based on experience and data regarding the expected traffic, the dispatcher decides about the number of sectors to open and the number of controllers in a particular sector during the considered time intervals.

Daily operational airspace division into sectors includes the following data:

- time period during which certain sectors are opened,
- the number of opened sectors,
- the sector type (i.e. which sectors are opened),
- the number of controllers assigned to particular sectors.

The problem of determining the airspace's operational division into sectors is extremely significant for the normal functioning of ATC. The significance is evident through the influence these divisions have on the ATC system. Two of the influences are important. First, daily operational division into sectors determines the ATC capacity (generally, the greater the number of sectors opened the greater the capacity of ATC and vice versa). This division also influences Central Flow Management Unit (CFMU) measures with the aim of air traffic flow management. Thus, on the basis of today for tomorrow, data about the number, type, capacity and open time of particular sectors during the day are sent to CFMU in Brussels. Second, daily operative divisions into sectors are the input data for determining the required number of controllers in a shift as well as the input data for determining the total number of controllers available when we take into consideration regulations concerning working time duration, the number of days off, etc. This operational sectorization is also useful to the dispatcher when assigning controllers to sectors during the shift.

The problem considered in this paper is defined in Section 2. The proposed problem solutions and numerical examples are presented in Section 3. Section 4 contains the direction of future research and conclusions.

## 2. Problem definition

On the basis of the preceding section, the problem which is considered in this paper can be defined as follows: For the planned or forecast data about the traffic characteristics (volume and traffic scenario) during the considered time period $\Delta t$, known capacity of individual or merged sectors during $\Delta t$ and predefined schedule of sector openings, determine the number of opened sectors during considered time period $\Delta t$ in such a way that safe, regular and expeditive air traffic is provided.

## 3. The proposed problem solutions

Traffic planning and control is connected with making certain decisions based on the input data (planned or forecasted). The input data are often imprecise or uncertain, and in some situations are the result of the decision-maker's subjective assessment. On the other hand, the systems considered are often so complex that it is impossible to determine all the relations that exist within them and thus it is impossible to determine a good model of a real system. Analysing a great number of variables under the condition when the variables are defined by imprecise data and when the functional relations among them cannot be determined, becomes very complicated and practically impossible using classical mathematical models.

One of the methods suitable for the consideration of uncertainty and subjectivity is based on fuzzy logic [1–5]. Models based on fuzzy logic consist of "If–Then" rules. The main idea is to develop a model which simulates the decision-making process without determining the exact functional relations between input and output variables.

This paper proposes two models based on fuzzy logic to solve the above-mentioned problem. In the first model fuzzy rules are defined by an expert while in the second model fuzzy rules are defined on the basis of input–output data pairs and expert knowledge.

### 3.1. Model I – Fuzzy algorithm developed on the basis of an expert's knowledge

The decision about the number of sectors to be opened is made by the dispatcher. The imposed question is: What are the most important input variables on the basis of which the dispatcher makes this decision?

After interviewing dispatchers it was concluded that the greatest influence on the number of sectors to open was the number of aircraft that overfly the considered airspace and the number of aircraft that take off (climb) or land (descend). Namely, an increase in the number of overflights produces an increase in

- the number of coordinations between controllers,
- the number of conversations between pilot and controller,
- the number of possible conflict situations (the separation between two aircraft is less than minimum) that the controller must detect in advance and take measures to solve.

The consequence of all this mentioned is an increase in the controller's work-load. The increase in the controller's work-load is acceptable up to a certain limit (the controller's capacity). For that reason, measures to reduce the controller's work-load are taken and one of them is opening new sectors.

Similar effects on the controller's work-load are also caused by an increase in the number of aircraft that climb after take off or descend for landing. It should be pointed out that in this case the controller's work-load is increased (with the appearance of a new aircraft) faster than in the case when the number of overflying aircraft is increased by one. The control of an aircraft that takes off or lands is a more difficult task for the controller than monitoring overflying aircraft (which usually do not change the flight level) because of frequent altitude changes.

Considering the above, "the number of overflights" and "the number of $T+L$" (the total number of take off and landing aircraft) during considered time period $\Delta t$ are taken for the input variables in this paper.

Data about the number of overflights and the number of take offs and landings in Yugoslavia airspace are obtained from strips. The data refer to the winter season in 1997. Data about the number of overflights and the number of take off and landing aircraft during periods of $\Delta t = 30$ min are obtained by the statistical processing of data. The number of take off and landing aircraft also includes the aircraft that take off or land at neighbouring countries' airports as well. This has been done due to the fact that these aircraft change their altitude during flight through Yugoslav airspace.

Having analysed the data obtained, the limits for the values of input fuzzy variables were determined (30-min intervals). The values of "the number of overflights" fuzzy input variable are between 0 and 20 aircraft, and the values of "the number of $T+L$" fuzzy input variable are between 0 and 15 aircraft.

On the basis of interviews with dispatchers, available data and numerous experiments which were made, it was concluded that "the number of overflights" and "the number of $T+L$" fuzzy input variables can be defined as follows: "very small", "small", "medium", "high" and "very high". A graphic presentation of these fuzzy input variables with the corresponding membership function values is presented in Fig. 1.

The output variable is defined as the preference to open a certain number of sectors (in the following text "division type"). The preferences are defined as: the preference of 1 sector open – "division type 1", the preference of 2 sectors open – "division type 2" and the preference of 3 sectors open – "division type 3". The three above-mentioned output variables have been taken into consideration because during the considered time period the number of opened sectors was between 1 and 3. A similar approach can be found in paper [3].

The preference value equal to one for the output variable, for example "division type 2", means that the dispatcher is absolutely sure that he has to open two sectors, zero means that he is absolutely sure that he will not open two sectors. Preference values between zero and one correspond to the strength of the dispatcher's willingness to open two sectors.

The preference of the "division type" output fuzzy variable is defined as "small", "medium" and "high" for all three fuzzy variables: "division type 1", "division type 2" and "division type 3". A graphic presentation of the output fuzzy variable "division type 1"

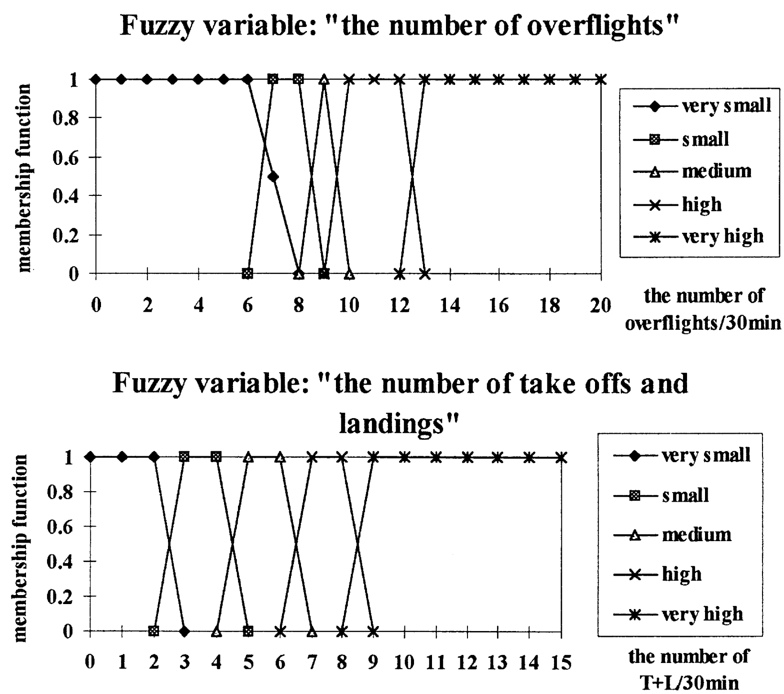


Fig. 1. Graphic presentation of the input fuzzy variables.

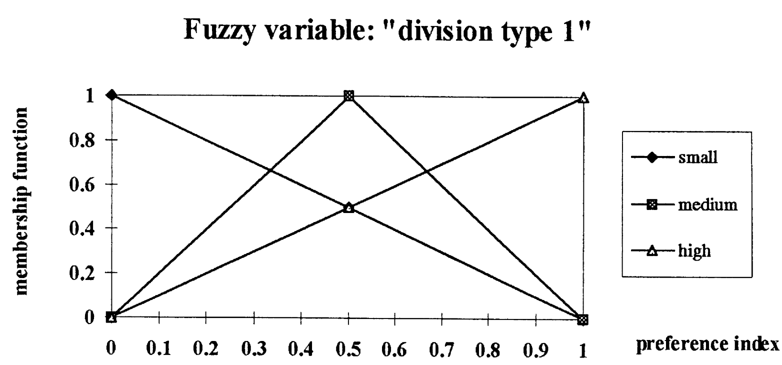


Fig. 2. Graphic presentation of the output fuzzy variable.

is presented in Fig. 2. The graphic presentation of the output fuzzy variable "division type 2" and "division type 3" is same as for variable "division type 1".

The algorithm proposed in this section to determine the required number of opened sectors ("division type") consists of a fuzzy rule base obtained on the basis of expert knowledge. For each of the mentioned output fuzzy variables a separate fuzzy rule base matrix has been formed (Figs. 3–5).

The rules are formed for sector capacity defined as a maximum 15 strips simultaneously present in a sector.

Having passed the input variables through the defined rules, the preference index values for "division type 1", "division type 2" and "division type 3" are obtained. Defuzzification is performed by accepting the value of the preference index which represents the centre of gravity. The maximum of the three preference index values is used

| No. OF OVERFLIGHTS | | | | | | |
|---|---|---|---|---|---|---|
| | **very high** | medium | small | small | small | small |
| | **high** | medium | small | small | small | small |
| | **medium** | high | medium | small | small | small |
| | **small** | high | high | medium | small | small |
| | **very small** | high | high | high | small | small |
| | | **very small** | **small** | **medium** | **high** | **very high** |
| | | **THE NUMBER OF TAKE OFFS AND LANDINGS** | | | | |

Fig. 3. The fuzzy rule base matrix for "division type 1".

| No. OF OVERFLIGHTS | | | | | | |
|---|---|---|---|---|---|---|
| | **very high** | medium | high | high | high | small |
| | **high** | medium | high | high | high | small |
| | **medium** | small | medium | high | high | small |
| | **small** | small | small | medium | high | small |
| | **very small** | small | small | small | high | medium |
| | | **very small** | **small** | **medium** | **high** | **very high** |
| | | **THE NUMBER OF TAKE OFFS AND LANDINGS** | | | | |

Fig. 4. The fuzzy rule base matrix for "division type 2".

| No. OF OVERFLIGHTS | | | | | | |
|---|---|---|---|---|---|---|
| | very high | small | small | small | medium | high |
| | high | small | small | small | small | high |
| | medium | small | small | small | small | high |
| | small | small | small | small | small | high |
| | very small | small | small | small | small | high |
| | | **very small** | **small** | **medium** | **high** | **very high** |
| | | **THE NUMBER OF TAKE OFFS AND LANDINGS** | | | | |

Fig. 5. The fuzzy rule base matrix for “division type 3”.

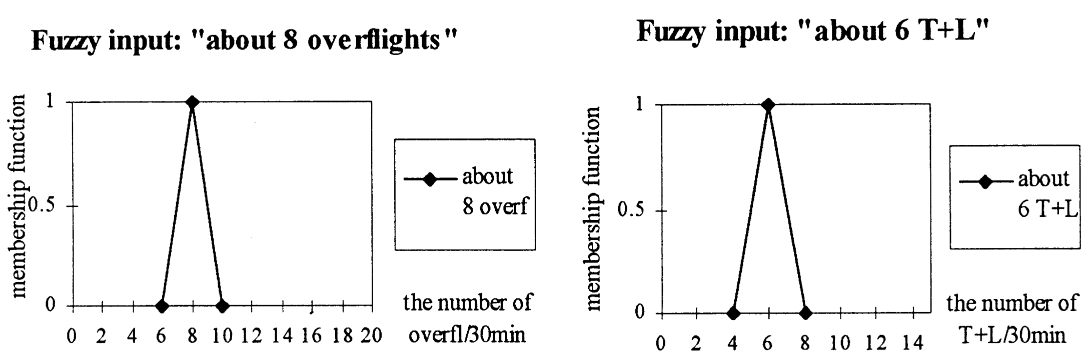


Fig. 6. Fuzzy input values.

as the value of the “division type” output variable. Namely, if for certain inputs the maximum preference index value is obtained for fuzzy variable $k$, then “division type $k$” is accepted as the required number of opened sectors, $k \in \{1,2,3\}$.

If the same preference index value is obtained for two fuzzy rule bases, the one “demanding” the greater number of sectors is accepted (a conservative approach due to safety precautions).

### 3.1.1. *Numerical example*

The values of input variables can be deterministic or fuzzy. In this paper the values of input variables are represented as triangular fuzzy numbers because of the uncertainty concerning the accuracy of the input variable. This uncertainty is always present because of the possible delays of flights, cancellation of flights, etc. Namely, the values of input fuzzy variables are obtained by forecasting the following few hours or following day, from the available data or on the basis of the dispatcher’s experience. Fig. 6 presents “about 8 overflights” and “about 6 T + L” fuzzy input numbers. The values of input variables presented in Table 1 (columns 2 and 3) are fuzzy numbers. The fuzzy logic algorithm is run with these.

Having passed through the fuzzy logic algorithm, output fuzzy variable values are obtained for all the $7 \times 48$ intervals (7 days $\times$ 48 intervals of 30 min).

Table 1
Results for Saturday, 08.02.1997

| Time | Fuzzy no. of overfl. | Fuzzy no. of $t/1$ | Division type 1 | Division type 2 | Division type 3 | No. of sect. (model) | No. of sect. (sample) |
|---|---|---|---|---|---|---|---|
| 00:00–00:30 | 5 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 00:30–01:00 | 4 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 01:00–01:30 | 0 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 01:30–02:00 | 4 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 02:00–02:30 | 3 | 2 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 02:30–03:00 | 6 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 03:00–03:30 | 12 | 1 | 0.50 | 0.50 | 0.33 | 2 | 1 |
| 03:30–04:00 | 8 | 0 | 0.64 | 0.36 | 0.33 | 1 | 1 |
| 04:00–04:30 | 7 | 2 | 0.64 | 0.36 | 0.33 | 1 | 1 |
| 04:30–05:00 | 7 | 4 | 0.58 | 0.42 | 0.33 | 1 | 1 |
| 05:00–05:30 | 3 | 1 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 05:30–06:00 | 10 | 3 | 0.47 | 0.53 | 0.33 | 2 | 1 |
| 06:00–06:30 | 5 | 2 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 06:30–07:00 | 7 | 4 | 0.58 | 0.42 | 0.33 | 1 | 1 |
| 07:00–07:30 | 5 | 2 | 0.67 | 0.33 | 0.33 | 1 | 2 |
| 07:30–08:00 | 4 | 9 | 0.33 | 0.57 | 0.53 | 2 | 2 |
| 08:00–08:30 | 9 | 4 | 0.50 | 0.50 | 0.33 | 2 | 2 |
| 08:30–09:00 | 14 | 3 | 0.39 | 0.61 | 0.33 | 2 | 2 |
| 09:00–09:30 | 11 | 3 | 0.42 | 0.58 | 0.33 | 2 | 2 |
| 09:30–10:00 | 8 | 3 | 0.58 | 0.42 | 0.33 | 1 | 2 |
| 10:00–10:30 | 6 | 6 | 0.53 | 0.47 | 0.33 | 1 | 2 |
| 10:30–11:00 | 7 | 4 | 0.58 | 0.42 | 0.33 | 1 | 2 |
| 11:00–11:30 | 8 | 5 | 0.50 | 0.50 | 0.33 | 2 | 3 |
| 11:30–12:00 | 4 | 11 | 0.33 | 0.50 | 0.67 | 3 | 3 |
| 12:00–12:30 | 8 | 4 | 0.53 | 0.47 | 0.33 | 1 | 3 |
| 12:30–13:00 | 8 | 7 | 0.44 | 0.56 | 0.39 | 2 | 3 |
| 13:00–13:30 | 16 | 8 | 0.33 | 0.53 | 0.55 | 3 | 3 |
| 13:30–14:00 | 12 | 9 | 0.33 | 0.47 | 0.53 | 3 | 3 |
| 14:00–14:30 | 17 | 10 | 0.33 | 0.39 | 0.64 | 3 | 3 |
| 14:30–15:00 | 6 | 1 | 0.67 | 0.33 | 0.33 | 1 | 3 |
| 15:00–15:30 | 12 | 6 | 0.33 | 0.67 | 0.39 | 2 | 2 |
| 15:30–16:00 | 10 | 7 | 0.36 | 0.61 | 0.39 | 2 | 2 |
| 16:00–16:30 | 5 | 10 | 0.33 | 0.52 | 0.61 | 3 | 2 |
| 16:30–17:00 | 8 | 3 | 0.58 | 0.42 | 0.33 | 1 | 2 |
| 17:00–17:30 | 3 | 5 | 0.61 | 0.39 | 0.33 | 1 | 2 |
| 17:30–18:00 | 4 | 6 | 0.53 | 0.47 | 0.33 | 1 | 2 |
| 18:00–18:30 | 8 | 0 | 0.64 | 0.36 | 0.33 | 1 | 2 |
| 18:30–19:00 | 6 | 4 | 0.61 | 0.39 | 0.33 | 1 | 2 |
| 19:00–19:30 | 3 | 5 | 0.61 | 0.39 | 0.33 | 1 | 1 |
| 19:30–20:00 | 6 | 2 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 20:00–20:30 | 5 | 3 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 20:30–21:00 | 2 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 21:00–21:30 | 1 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 21:30–22:00 | 4 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 22:00–22:30 | 3 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 22:30–23:00 | 2 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 23:00–23:30 | 5 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |
| 23:30–24:00 | 0 | 0 | 0.67 | 0.33 | 0.33 | 1 | 1 |

Applying defuzzification, the values of the required number of open sectors are obtained. The results for Saturday, 08.02.1997 are presented in Table 1 (column – Number of sectors (model)) together with data on the number of sectors which were really opened during the time period considered (column – Number of sectors (sample)).

At first sight it can be concluded that there are significant differences between the column – Number of sectors (model) and the column – Number of sectors (sample). Comparing the results it turns out that they correspond to 62.5% of the cases. However, taking into consideration the facts that

- opening and closing sectors last a certain time,
- opening and closing sectors are performed with step 1 ($1 \rightarrow 2 \rightarrow 3$ sectors, or $3 \rightarrow 2 \rightarrow 1$),

it is clear that the dispatcher makes the decision about the required number of open sectors at a certain time period only after having analysed the data about the expected traffic in the intervals before and after the considered period. Thus, for example, if the requested number of opened sectors at isolated, but successive periods of 30 min, are the following: $2, 2, 1, 1, 2, 2, 1, 2$ – the dispatcher will in this case keep two sectors open all the time. Due to practical reasons the dispatcher will not open and close sectors for short time intervals.

Taking into consideration the facts mentioned, an algorithm which simulates the dispatcher while making a decision about opening/closing sectors was developed.

The input data to the algorithm was sequence $\{P_i\}$, $i = 1, \ldots, 48$ where $P_i$ represents the number of opened sectors at an isolated period $i$ obtained by the fuzzy logic model. The final number of opened sectors at the considered time period $i$ ($P_i'$), is determined on the basis of the following data:

- the number $P_i$ of opened sectors at the considered time period,
- the number of opened sectors accepted for the previous period ($P_{i-1}$),
- the number of opened sectors in the following $k$ periods ($P_{i+1}, \ldots, P_{i+k}$).

The first element and the last $k$ elements of the sequence $\{P_i\}$ cannot be considered by the algorithm like the other ones. These elements are considered separately. The first element is considered as a separate case since there is no preceding element and for the last $k$ elements there are no following $k$ elements. For the first element value $P_1'$ is determined based on the value of $k$ following elements ($P_2, P_3, \ldots, P_{1+k}$) while for the last $k$ elements value $P_i'$, $i = 48-(k+1), \ldots, 48$ is determined based on the value of $k$ preceding elements ($P_{i-1}, P_{i-2}, \ldots, P_{i-k}$).

The proposed algorithm to calculate the final number of opened sectors $P_i'$, $i = 2, \ldots, 48-k$, which simulates the dispatcher's decision-making process when considering successive $k = 3$ time periods, work as follows:

1. IF for considered time period $i$, $P_i = P_j$ for all elements $P_j$, $j = i-1, \ldots, i+k$ THEN $P_i' = P_i$.
2. IF for considered time period $i$, $P_i' = P_j + 1$ for less than $k$ elements $P_j$ ($j = i-1, \ldots, i+k$) THEN $P_i' = P_i$.
3. IF for considered time period $i$, $P_i = P_j + 1$ for all elements $P_j$ ($j = i-1, \ldots, i+k$) THEN $P_i' = P_i - 1$.
4. IF for considered time period $i$, $P_i = P_j + 2$ for at least one element $P_j$ ($j = i-1, \ldots, i+k$) THEN $P_i' = P_i$.
5. IF for considered time period $i$, $P_i = P_j - 1$ for at least one element $P_j$ ($j = i-1, \ldots, i+k$) THEN $P_i' = P_i + 1$.
6. IF for considered time period $i$, $P_i = P_j - 2$ for at least one element $P_j$ ($j = i-1, \ldots, i+k$) THEN $P_i' = P_i + 2$.

A similar procedure is applied when considering the first element $P_1$ and the last $k$ elements $P_i$, $i = 48 - (k+1), \ldots, 48$.

Having passed through the algorithm the final required number of opened sectors $P_i'$, $i = 1, \ldots, 48$ is obtained. The results obtained for Saturday 08.02 are presented in Table 2 (column – Number of sectors (algorithm)).

By comparing the column – Number of sectors (algorithm) and the column – Number of sectors (sample) from Table 2, it can be concluded that after using the algorithm the correspondence has significantly increased (for Saturday the obtained correspondence is 79.17%). The reasons for not obtaining even better correspondence between the results are due to the fact that

- a conservative approach was taken in determining the division type at the time interval from 05:30 to 07:00; the algorithm gave division type 2 instead of division type 1.

Table 2
Results for Saturday, 08.02.1997

| Time | No. of sectors (model) | No. of sectors (algorithm) | No. of sectors (sample) |
|---|---|---|---|
| 00:00–00:30 | 1 | 1 | 1 |
| 00:30–01:00 | 1 | 1 | 1 |
| 01:00–01:30 | 1 | 1 | 1 |
| 01:30–02:00 | 1 | 1 | 1 |
| 02:00–02:30 | 1 | 1 | 1 |
| 02:30–03:00 | 1 | 1 | 1 |
| 03:00–03:30 | 2 | 1 | 1 |
| 03:30–04:00 | 1 | 1 | 1 |
| 04:00–04:30 | 1 | 1 | 1 |
| 04:30–05:00 | 1 | 1 | 1 |
| 05:00–05:30 | 1 | 1 | 1 |
| 05:30–06:00 | 2 | 2 | 1 |
| 06:00–06:30 | 1 | 2 | 1 |
| 06:30–07:00 | 1 | 2 | 1 |
| 07:00–07:30 | 1 | 2 | 2 |
| 07:30–08:00 | 2 | 2 | 2 |
| 08:00–08:30 | 2 | 2 | 2 |
| 08:30–09:00 | 2 | 2 | 2 |
| 09:00–09:30 | 2 | 2 | 2 |
| 09:30–10:00 | 1 | 2 | 2 |
| 10:00–10:30 | 1 | 2 | 2 |
| 10:30–11:00 | 1 | 2 | 2 |
| 11:00–11:30 | 2 | 2 | 3 |
| 11:30–12:00 | 3 | 3 | 3 |
| 12:00–12:30 | 1 | 3 | 3 |
| 12:30–13:00 | 2 | 3 | 3 |
| 13:00–13:30 | 3 | 3 | 3 |
| 13:30–14:00 | 3 | 3 | 3 |
| 14:00–14:30 | 3 | 3 | 3 |
| 14:30–15:00 | 1 | 2 | 3 |
| 15:00–15:30 | 2 | 2 | 2 |
| 15:30–16:00 | 2 | 2 | 2 |
| 16:00–16:30 | 3 | 2 | 2 |
| 16:30–17:00 | 1 | 1 | 2 |
| 17:00–17:30 | 1 | 1 | 2 |
| 17:30–18:00 | 1 | 1 | 2 |
| 18:00–18:30 | 1 | 1 | 2 |
| 18:30–19:00 | 1 | 1 | 2 |
| 19:00–19:30 | 1 | 1 | 1 |
| 19:30–20:00 | 1 | 1 | 1 |
| 20:00–20:30 | 1 | 1 | 1 |
| 20:30–21:00 | 1 | 1 | 1 |
| 21:00–21:30 | 1 | 1 | 1 |
| 21:30–22:00 | 1 | 1 | 1 |
| 22:00–22:30 | 1 | 1 | 1 |
| 22:30–23:00 | 1 | 1 | 1 |
| 23:00–23:30 | 1 | 1 | 1 |
| 23:30–24:00 | 1 | 1 | 1 |

- the dispatcher made a "delay" with closing the sectors (the intervals 14:30–15:00 and 16:30–19:00).

Considering the above mentioned, it can be concluded that the proposed model has given quite satisfactory results.

### 3.2. Model II – generating fuzzy rules from a sample

In the last few years a great number of papers have presented methods for generating the fuzzy rule base by combining two type of rules:

1. Fuzzy rules generated from the pairs of available input–output data (numerical rules).
2. Fuzzy rules based on expert knowledge (linguistic rules).

One of most important papers in this field is paper [6].

Let us suppose that there is a problem as follows: For a complex control system, for which there is no defined mathematical model and in which man has an essential part in the decision-making process, define a model to replace the decision maker in cases when the following information is available:

1. An expert's (decision maker's) experience (usually expressed through the linguistic "If–Then" rules showing the implemented actions in particular situations).
2. Pairs of input–output data obtained by recording an expert's work, i.e. numerical data of the form

$$(x_1^{(1)}, x_2^{(1)}, y_1^{(1)}), (x_1^{(2)}, x_2^{(2)}, y_1^{(2)}), \ldots,$$

where $x_1^{(i)}$ and $x_2^{(i)}$ are inputs to the system and, $y^{(i)}$ is an output of the system.

Separately considered, this information is often insufficient to form a good model. Namely, when experts express their experience through linguistic rules some of the information gets lost. On the other hand, the information obtained from the pairs of input–output data is often insufficient, since the data cannot include all the situations in which the system can be found. Wang and Mendel [6] proposed a method to form the fuzzy rule base using both numerical data and an expert's experience, i.e. a combination of numerical and linguistic rules. With this approach much better results can be expected compared to the results obtained using only one rule type.

The Wang and Mendel method [6] to generate the fuzzy rule base consists of five steps:

*Step* 1: From numerical input and output data determine fuzzy input and output variable domains and the corresponding shape of membership functions.

*Step* 2: Generate fuzzy rules from the given pairs of input–output data in the following way:

First, determine the membership function values for the given input $(x_1^{(i)}, x_2^{(i)})$ and output $(y^{(i)})$ values.

Second, assign the value $x_1^{(i)}$, $x_2^{(i)}$ and $y^{(i)}$ to those fuzzy numbers for which the maximum membership function value is obtained. Namely, if for $x_1^{(i)}$ the membership function value is maximal $(\mu_{\max})$ for fuzzy number $A$, for $x_2^{(i)}$, $\mu_{\max}$ is for fuzzy number $B$ and for $y^{(i)}$ $\mu_{\max}$ is for fuzzy number $C$, then for given input–output data pair a rule is to be formed as follows:

RULE $i$: If $(x_1^{(i)} = A$ and $x_2^{(i)} = B)$

Then $y^{(i)} = C$

*Step* 3: Assign a degree to each rule.

Generally, there is a great number of input–output data pairs, so a great number of “If–Then” rules will be inherently formed. There is a high probability that, among the generated rules, a certain number of conflict rules will appear (rules which for the same IF part gives different THEN parts). There are several ways to choose the “relevant” rule from a group of conflict rules. Some of them are the following:

1. Assign a certain degree to each of the generated rules and accept such rule from the conflict group which has the maximum degree. Thus the total number of rules is considerably decreased. For Rule $i$ (If $x_1^{(i)} = A$ and $x_2^{(i)} = B$ Then $y^{(i)} = C$), the degree is defined as

$$D(\text{RULE } i) = \mu_A(x_1^{(i)}) \cdot \mu_B(x_2^{(i)}) \cdot \mu_C(y^{(i)}),$$

where $\mu_A(x_1^{(i)})$ is the membership function of $x_1^{(i)}$ to fuzzy number $A$, $\mu_B(x_2^{(i)})$ is the membership function of $x_2^{(i)}$ to fuzzy number $B$, and $\mu_C(y^{(i)})$ is the membership function of $y^{(i)}$ to fuzzy number $C$.

2. Assign to each pair of input–output data a degree representing an expert’s opinion about the significance of the given data pair. Certain data pairs are very significant and essential for determining fuzzy rules, while some of them are usually obtained by mistake – so-called “wild data”. If to a data pair $(x_1^{(i)}, x_2^{(i)}; y^{(i)})$ a degree $\mu^{(i)}$ is assigned by an expert, then the rule degree is defined as

$$D(\text{RULE } i) = \mu_A(x_1^{(i)}) \cdot \mu_B(x_2^{(i)}) \cdot \mu_C(y^{(i)}) \cdot \mu^{(i)},$$

i.e. the rule degree is defined as the multiplication of membership function values of corresponding fuzzy numbers of the rule components and the degree representing an expert opinion for the generated rule.

*Step 4*: Create a combined fuzzy rule base. The rule base is given in the form of a matrix. The matrix cells are filled in the following way:

1. If more than one rule is generated from the pair of input–output data for a certain cell, accept the rule with the maximum degree.

2. If a rule for a certain cell is not generated from the pair of input–output data, that cell is filled by the rule obtained based on expert knowledge.

*Step* 5: Determine the mapping of the inputs into the output value on the basis of the combined fuzzy rule base, i.e. implement defuzzification.

### *3.2.1. Numerical example*

After numerous experiments with the available data the shape of the membership functions corresponding to the input fuzzy variables and output fuzzy variable are determined. “The number of overflights” and “the number of T + L” input fuzzy variables are defined as “very very small”, “very small”, “small”, “medium”, “high”, “very high”. A graphic presentation of these fuzzy variables with the corresponding membership function values is presented in Fig. 7.

“The number of sectors” output fuzzy variable, denoting the required number of open sectors (division type), is defined as: “1 sector”, “2 sectors”, “3 sectors”. These fuzzy numbers are presented in Fig. 8.

On the basis of the input–output data pairs, “If–Then” rules are generated as described in Step 2 (Section 3.2). A great number of rules are generated and a considerable number of conflict rules are obtained.

One of the ways to resolve the conflict rules is assigning a degree to each rule as described in Step 3/1 (Section 3.2). However, this procedure did not give satisfactory results, i.e. conflict rules were not resolved for two reasons:

- first, the input and output fuzzy variables are defined in such a way that the rule degree is very often equal to 1,

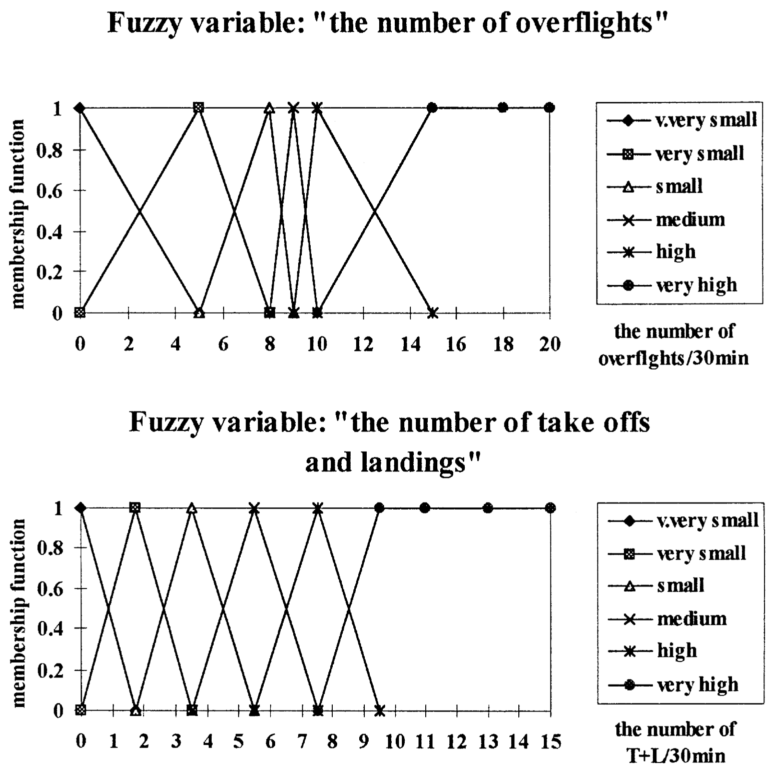


Fig. 7. Graphic presentation of the input fuzzy variables.

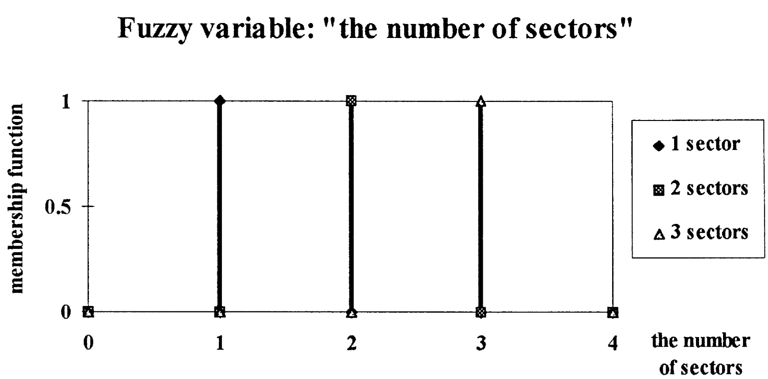


Fig. 8. Graphic presentation of the fuzzy variable "the number of sectors".

| No. OF OVERFLIGHTS | THE NUMBER OF TAKE OFFS AND LANDINGS: v.very small | very small | small | medium | high | very high |
|---|---|---|---|---|---|---|
| very high | 1 sector 0.0100 | | 2 sectors 0.6818 | 2 sectors 0.6944 | 3 sectors 0.0083 | 3 sectors 1.0000 |
| high | 1 sector 0.8333 | 1 sector 0.3810 | 2 sectors 0.6944 | 2 sectors 0.8333 | 2 sectors 0.8333 | 3 sectors 1.0000 |
| medium | 1 sector 1.0000 | 3 sectors 0.0091 | 2 sectors 0.0083 | 2 sectors 0.8333 | 2 sectors 0.8333 | 3 sectors 0.8333 |
| small | 1 sector 1.0000 | 1 sector 0.9091 | 1 sector 0.8182 | 2 sectors 0.0083 | 2 sectors 0.8333 | 3 sectors 1.0000 |
| very small | 1 sector 1.0000 | 1 sector 0.9091 | 1 sector 0.8182 | 1 sector 0.8333 | 2 sectors 0.8333 | 3 sectors 0.8333 |
| v.very small | 1 sector 1.0000 | 1 sector 0.7273 | 1 sector 0.6545 | 1 sector 0.6667 | 2 sectors 0.5000 | |

Fig. 9. The initial fuzzy rule base matrix.

- second, even if the membership functions of fuzzy numbers are defined with the other shape, for rules formed from certain input–output data pairs which represent so-called "wild data", a degree higher can be obtained than the degree for "correct" rules. Therefore it was necessary to include an expert's knowledge as well, which has been done in the following way:

To each pair of input–output data a degree $\mu^{(i)}$ was assigned. It represents an expert's opinion about the given data pair's significance for generating the rule. For pairs representative for generating the rules the degree assigned was $\mu^{(i)} = 1$, and for pairs which for certain entries had "wrong" exits the degree assigned was $\mu^{(i)} = 0.01$. Based on this procedure the assigned rule degree for each rule generated from the pair of input–output data was defined as

$$D(\text{RULE } i) = \mu_A(x_1^{(i)}) \cdot \mu_B(x_2^{(i)}) \cdot \mu_C(y^{(i)}) \cdot \mu^{(i)}.$$

Based on the above-described procedure, the initial fuzzy rule base (generated from a sample) was formed. The rule base with corresponding degrees is given in the form of a matrix (Fig. 9). Shaded cells in Fig. 9 represent cells for which no rule is generated or cells for which rule degree is $\leqslant 0.01$.

The final content of the fuzzy rule base is obtained in the following way:

1. For the input pairs: "very very small number of overflights" – "very high number of T + L" and "very high number of overflights" – "very small number of T + L", the cells of the rule base matrix have remained unfilled. That means that the data used to generate the rules had no such input pairs. These matrix cells are filled in on the basis of an expert's knowledge.

2. Certain cells of the initial rule base matrix are filled with rules which have the degree $\leqslant 0.01$. This implies that these rules are formed from "wild" input–output data pairs. These rules cannot be accepted, so the matrix cells with such rules are also filled on the basis of an expert's knowledge.

The final fuzzy rule base matrix is presented in Fig. 10.

Passing the input data through the defined fuzzy logic algorithm and after defuzzification the value of the output fuzzy variable is obtained. The obtained value of "the number of sectors" output variable, i.e. the required number of opened sectors is presented in the column "Number of sectors (model)" – Table 3. The values obtained present the input into the algorithm (described in Section 3.1.1) which simulates a

| No. OF OVERFLIGHTS | | | | | | | |
|---|---|---|---|---|---|---|---|
| | very high | 2 sectors | 2 sectors | 2 sectors | 2 sectors | 2 sectors | 3 sectors |
| | high | 1 sector | 1 sector | 2 sectors | 2 sectors | 2 sectors | 3 sectors |
| | medium | 1 sector | 1 sector | 1 sector | 2 sectors | 2 sectors | 3 sectors |
| | small | 1 sector | 1 sector | 1 sector | 1 sector | 2 sectors | 3 sectors |
| | very small | 1 sector | 1 sector | 1 sector | 1 sector | 2 sectors | 3 sectors |
| | v.very small | 1 sector | 1 sector | 1 sector | 1 sector | 2 sectors | 3 sectors |
| | | **v.very small** | **very small** | **small** | **medium** | **high** | **very high** |
| | | **THE NUMBER OF TAKE OFF AND LANDINGS** | | | | | |

Fig. 10. The final fuzzy rule base matrix.

dispatcher's decision-making process when deciding about the number of opened sectors. The values of the required number of open sectors obtained by this algorithm are presented in the column "Number of sectors (algorithm)" – Table 3.

The results for Saturday 08.02.1997 are presented in Table 3, together with data about the number of sectors opened at the considered time intervals in the sample (column – "Number of sectors (sample))".

Comparing the column "Number of sectors (algorithm)" and the column "Number of sectors (sample)" the obtained correspondence is in 85.14% of the cases.

As with the previous model (Model I), in Model II there would have been better correspondence (95.92%) if a conservative approach had not been applied to the model (referring to the period from 05:30 to 07:00) and if the dispatcher had not delayed with closing the sectors (referring to the period 18:00 to 19:00, Table 3).

In Table 4 the results obtained from the application of Models I and II are given together with the data about the number of opened sectors in the sample for Saturday 08.02.1997. It can be seen in Table 4 that the results obtained by Model II are a little bit better than by Model I. Namely, in Model II, the obtained correspondence between the model results and the number of opened sectors in the sample is 85.14%, while the correspondence achieved by Model I is 79.16%. Similar results were obtained for the other weekdays.

## 4. Conclusion

This paper proposed models based on the application of fuzzy logic to solve the daily sectorization problem, i.e. the problem of determining the distribution of the required number of sectors open during the day for a given airway network in a considered airspace, given airspace sectorization and planned (forecast) traffic characteristics.

Two models are proposed to solve the defined problem.

In Model I (the proposed fuzzy logic algorithm is defined on the basis of an expert's knowledge) the criteria on the basis of which the dispatcher makes the decision about the required number of opened sectors are defined and fuzzy rules are generated on the basis of an expert's knowledge.

In Model II the proposed fuzzy logic algorithm is based on the fuzzy rule base obtained from a sample of

Table 3
Results for Saturday, 08.02.1997

| Time | No. of overflights | No. of T + L | No. of sectors (model) | No. of sectors (algorithm) | No. of sectors (sample) |
|---|---|---|---|---|---|
| 00:00–00:30 | 5 | 0 | 1 | 1 | 1 |
| 00:30–01:00 | 4 | 0 | 1 | 1 | 1 |
| 01:00–01:30 | 0 | 0 | 1 | 1 | 1 |
| 01:30–02:00 | 4 | 0 | 1 | 1 | 1 |
| 02:00–02:30 | 3 | 2 | 1 | 1 | 1 |
| 02:30–03:00 | 6 | 0 | 1 | 1 | 1 |
| 03:00–03:30 | 12 | 1 | 2 | 1 | 1 |
| 03:30–04:00 | 8 | 0 | 1 | 1 | 1 |
| 04:00–04:30 | 7 | 2 | 1 | 1 | 1 |
| 04:30–05:00 | 7 | 4 | 1 | 1 | 1 |
| 05:00–05:30 | 3 | 1 | 1 | 1 | 1 |
| 05:30–06:00 | 10 | 3 | 2 | 2 | 1 |
| 06:00–06:30 | 5 | 2 | 1 | 2 | 1 |
| 06:30–07:00 | 7 | 4 | 1 | 2 | 1 |
| 07:00–07:30 | 5 | 2 | 1 | 2 | 2 |
| 07:30–08:00 | 4 | 9 | 3 | 2 | 2 |
| 08:00–08:30 | 9 | 4 | 2 | 2 | 2 |
| 08:30–09:00 | 14 | 3 | 2 | 2 | 2 |
| 09:00–09:30 | 11 | 3 | 2 | 2 | 2 |
| 09:30–10:00 | 8 | 3 | 1 | 2 | 2 |
| 10:00–10:30 | 6 | 6 | 2 | 2 | 2 |
| 10:30–11:00 | 7 | 4 | 1 | 2 | 2 |
| 11:00–11:30 | 8 | 5 | 2 | 2 | 3 |
| 11:30–12:00 | 4 | 11 | 3 | 3 | 3 |
| 12:00–12:30 | 8 | 4 | 2 | 3 | 3 |
| 12:30–13:00 | 8 | 7 | 2 | 3 | 3 |
| 13:00–13:30 | 16 | 8 | 3 | 3 | 3 |
| 13:30–14:00 | 12 | 9 | 3 | 3 | 3 |
| 14:00–14:30 | 17 | 10 | 3 | 3 | 3 |
| 14:30–15:00 | 6 | 1 | 1 | 2 | 3 |
| 15:00–15:30 | 12 | 6 | 2 | 2 | 2 |
| 15:30–16:00 | 10 | 7 | 2 | 2 | 2 |
| 16:00–16:30 | 5 | 10 | 3 | 2 | 2 |
| 16:30–17:00 | 8 | 3 | 1 | 2 | 2 |
| 17:00–17:30 | 3 | 5 | 1 | 2 | 2 |
| 17:30–18:00 | 4 | 6 | 2 | 2 | 2 |
| 18:00–18:30 | 8 | 0 | 1 | 1 | 2 |
| 18:30–19:00 | 6 | 4 | 1 | 1 | 2 |
| 19:00–19:30 | 3 | 5 | 1 | 1 | 1 |
| 19:30–20:00 | 6 | 2 | 1 | 1 | 1 |
| 20:00–20:30 | 5 | 3 | 1 | 1 | 1 |
| 20:30–21:00 | 2 | 0 | 1 | 1 | 1 |
| 21:00–21:30 | 1 | 0 | 1 | 1 | 1 |
| 21:30–22:00 | 4 | 0 | 1 | 1 | 1 |
| 22:00–22:30 | 3 | 0 | 1 | 1 | 1 |
| 22:30–23:00 | 2 | 0 | 1 | 1 | 1 |
| 23:00–23:30 | 5 | 0 | 1 | 1 | 1 |
| 23:30–24:00 | 0 | 0 | 1 | 1 | 1 |

Table 4
Results of Models I and II for Saturday, 08.02.1997

| Time | No. of overfl. | No. of T+L | No. of sectors (sample) | No. of sectors (Model I) | No. of sec Model I (algorith) | No. of sectors (Model II) | No. of sec Model II (algorith) |
|---|---|---|---|---|---|---|---|
| 00:00–00:30 | 5 | 0 | 1 | 1 | 1 | 1 | 1 |
| 00:30–01:00 | 4 | 0 | 1 | 1 | 1 | 1 | 1 |
| 01:00–01:30 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 01:30–02:00 | 4 | 0 | 1 | 1 | 1 | 1 | 1 |
| 02:00–02:30 | 3 | 2 | 1 | 1 | 1 | 1 | 1 |
| 02:30–03:00 | 6 | 0 | 1 | 1 | 1 | 1 | 1 |
| 03:00–03:30 | 12 | 1 | 1 | 2 | 1 | 2 | 1 |
| 03:30–04:00 | 8 | 0 | 1 | 1 | 1 | 1 | 1 |
| 04:00–04:30 | 7 | 2 | 1 | 1 | 1 | 1 | 1 |
| 04:30–05:00 | 7 | 4 | 1 | 1 | 1 | 1 | 1 |
| 05:00–05:30 | 3 | 1 | 1 | 1 | 1 | 1 | 1 |
| 05:30–06:00 | 10 | 3 | 1 | 2 | 2 | 2 | 2 |
| 06:00–06:30 | 5 | 2 | 1 | 1 | 2 | 1 | 2 |
| 06:30–07:00 | 7 | 4 | 1 | 1 | 2 | 1 | 2 |
| 07:00–07:30 | 5 | 2 | 2 | 1 | 2 | 1 | 2 |
| 07:30–08:00 | 4 | 9 | 2 | 2 | 2 | 3 | 2 |
| 08:00–08:30 | 9 | 4 | 2 | 2 | 2 | 2 | 2 |
| 08:30–09:00 | 14 | 3 | 2 | 2 | 2 | 2 | 2 |
| 09:00–09:30 | 11 | 3 | 2 | 2 | 2 | 2 | 2 |
| 09:30–10:00 | 8 | 3 | 2 | 1 | 2 | 1 | 2 |
| 10:00–10:30 | 6 | 6 | 2 | 1 | 2 | 2 | 2 |
| 10:30–11:00 | 7 | 4 | 2 | 1 | 2 | 1 | 2 |
| 11:00–11:30 | 8 | 5 | 3 | 2 | 2 | 2 | 2 |
| 11:30–12:00 | 4 | 11 | 3 | 3 | 3 | 3 | 3 |
| 12:00–12:30 | 8 | 4 | 3 | 1 | 3 | 2 | 3 |
| 12:30–13:00 | 8 | 7 | 3 | 2 | 3 | 2 | 3 |
| 13:00–13:30 | 16 | 8 | 3 | 3 | 3 | 3 | 3 |
| 13:30–14:00 | 12 | 9 | 3 | 3 | 3 | 3 | 3 |
| 14:00–14:30 | 17 | 10 | 3 | 3 | 3 | 3 | 3 |
| 14:30–15:00 | 6 | 1 | 3 | 1 | 2 | 1 | 2 |
| 15:00–15:30 | 12 | 6 | 2 | 2 | 2 | 2 | 2 |
| 15:30–16:00 | 10 | 7 | 2 | 2 | 2 | 2 | 2 |
| 16:00–16:30 | 5 | 10 | 2 | 3 | 2 | 3 | 2 |
| 16:30–17:00 | 8 | 3 | 2 | 1 | 1 | 1 | 2 |
| 17:00–17:30 | 3 | 5 | 2 | 1 | 1 | 1 | 2 |
| 17:30–18:00 | 4 | 6 | 2 | 1 | 1 | 2 | 2 |
| 18:00–18:30 | 8 | 0 | 2 | 1 | 1 | 1 | 1 |
| 18:30–19:00 | 6 | 4 | 2 | 1 | 1 | 1 | 1 |
| 19:00–19:30 | 3 | 5 | 1 | 1 | 1 | 1 | 1 |
| 19:30–20:00 | 6 | 2 | 1 | 1 | 1 | 1 | 1 |
| 20:00–20:30 | 5 | 3 | 1 | 1 | 1 | 1 | 1 |
| 20:30–21:00 | 2 | 0 | 1 | 1 | 1 | 1 | 1 |
| 21:00–21:30 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 21:30–22:00 | 4 | 0 | 1 | 1 | 1 | 1 | 1 |
| 22:00–22:30 | 3 | 0 | 1 | 1 | 1 | 1 | 1 |
| 22:30–23:00 | 2 | 0 | 1 | 1 | 1 | 1 | 1 |
| 23:00–23:30 | 5 | 0 | 1 | 1 | 1 | 1 | 1 |
| 23:30–24:00 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |

input–output data pairs and rules based on an expert's knowledge.

Both proposed models were tested on a 7-day sample (the period from 07.02. to 13.02.1997). Based on the performed experiments the following conclusions have been made:

- both of the models give satisfactory results although the results obtained with Model II are a little bit better than those obtained with Model I,
- slight deviations of model results as regards the sample can still be reduced with further experimenting (model calibration),
- even a less experienced dispatcher can successfully use the proposed models in the process of determining the distribution of the number of opened sectors during the considered time interval (airspace daily sectorization) which was the main motivation behind this research.

The results obtained on the basis of the proposed models represent the input data for determining the total number of controllers needed, the number of controllers per shift and controllers' work schedule respecting the legal constraints as the number of days off, the duration of working hours, breaks during the shift, etc. The above-mentioned problems will be the subject of consideration in further research.

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